A New Approach for Calculating Position Domain Integrity Risk for Cycle Resolution in Carrier Phase Navigation Systems

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Abstract - **This paper describes a new theoretical approach to quantify the position-domain integrity risk for cycle ambiguity resolution problems. It is typically conservatively assumed that all incorrectly fixed cycle ambiguities cause hazardously large position errors. Although practical and previously used, this conservative assumption can unnecessarily limit navigation availability for applications with stringent requirements for accuracy and integrity. In response, a new method for calculating the fault free integrity risk for carrier phase navigation algorithms is developed. In this method we evaluate the impact of incorrect fixes in the position domain. The method is used to define tight upper bounds on integrity risk in carrier phase navigation systems that demand high availability. A mechanism to implement this method with partially-fixed cycle ambiguity vectors is also derived. The improvement in navigation availability using this method is quantified through covariance analysis simulated over a range of error model parameters.**

I. INTRODUCTION

The use of carrier phase measurements is necessary in high accuracy and high integrity navigation applications such as autonomous airborne refueling [1, 2] and autonomous shipboard landing [3, 4]. In navigation systems, fault free integrity risk is typically quantified as the probability that the position error exceeds predefined alert limits. In order to obtain centimeter level accuracy, carrier phase cycle ambiguities must be resolved. Over the last two decades, a multitude of research has been conducted in the area of cycle resolution and many different approaches have been developed to fix cycle ambiguities. A summary of the most frequently used methods is provided in [5]. Some of these solutions provide a measure of the probability of correct cycle estimation. In applications where integrity and accuracy requirements are stringent the cycle resolution problem can be very challenging.

In a method that has been previously used to account for integrity risk in cycle resolution, a threshold on the Probability of Incorrect Fix (*PIF*) is defined [6]. The PIF threshold must be lower than the allowable overall navigation integrity risk. During a typical navigation operation, only a subset of ambiguities can fixed with the required total PIF, leaving the rest as real numbers. This set of partially fixed ambiguities is used to estimate the position and the corresponding protection level. The protection level, defined as a statistical overbound for position errors consistent with the allowable integrity risk, is then compared to the position domain alert limits for the navigation operation. However, in this method it is implicitly and conservatively assumed that all incorrect integer ambiguity candidates will cause the position estimate errors to exceed the alert limits. Unfortunately, fixing a sufficient number of ambiguities to meet the tight accuracy requirements with a success rate that meets the integrity risk requirement is not always possible under all conditions [7]. In response, an innovative approach is developed to calculate the integrity risk by evaluating the impact of the incorrect integer candidates in the position domain. In addition, a mechanism to implement this method with a partially fixed solution in a navigation system is described.

The availability of carrier phase relative navigation can be greatly enhanced by increasing the *PIF* threshold [8]. However, as noted above, this probability is usually bounded by the integrity risk requirement. In this paper, we first revisit the previously used method where the *PIF* threshold is conservatively derived from the integrity risk requirement. Next, a theoretical derivation for a tighter upper bound on the integrity risk of ambiguity fix is formulated probabilistically, in which the effect of different sets of ambiguity candidates on the position error is evaluated. To further improve the performance using the new proposed method, we introduce a strategy to obtain partially fixed solutions. In this approach, a subset of ambiguities is fixed only if the resulting position domain integrity risk meets the requirements. The performance of the new method is initially quantified based on a simple geometry test-case. This is followed by an example application of the method to an autonomous shipboard landing navigation. Availability analysis over a wide range of error model parameters that were extracted from [7] is conducted to quantify the performance of the new method in a real application.

II. CONVENTIONAL METHOD FOR DERIVING THE ROBABILITY OF INCORRECT FIX FROM THE INTEGRITY RISK REQUIREMENT

This section presents the conventional method that has been derived in [6] which provides a formula for the relation between the *PIF* and the fault-free integrity requirement ($I_{H0\,req}$). To derive such a formula, it is easiest to start with the concept of the fault free Vertical Protection Level (VPL_{H0}). VPL_{H0} is defined as a statistical overbound where the probability of the vertical position estimate error ($|\hat{x}_v - x_v|$) exceeding VPL_{H0} equals $I_{H0\,req}$ (1).

$$P\{|\hat{x}_v - x_v| > VPL_{H0}\} = I_{H0\,req} \tag{1}$$

where,

- $P\{\blacksquare\}$: the probability of event '■'
- $\hat{x}_v$: vertical component of the estimated relative position vector
- x_v: vertical component of the true relative position vector

After fixing the ambiguities, two mutually exclusive and exhaustive events can be defined in relation to (1): a *correct fix* (CF) and an *incorrect fix* (IF) event. An IF includes all events where some or all ambiguities have been fixed incorrectly. A CF, on the other hand, includes only those events where no ambiguities are fixed incorrectly. Note that by the definition of a correct fix, if we choose not to fix any ambiguity, thereby leaving the cycle ambiguities floating, then the Probability of Correct Fix (*PCF*) is equal to 1. At the same time, if we choose to fix some or all ambiguities, we expect *PCF* to be less than 1. Using the law of total probability, equation (1) can be expanded to,

$$\begin{aligned} P\{|\hat{x}_v - x_v| > VPL_{H0}\} &= P\{|\hat{x}_v - x_v| > VPL_{H0} \mid CF\}\, PCF \\ &\quad + P\{|\hat{x}_v - x_v| > VPL_{H0} \mid IF\}\; PIF \end{aligned} \tag{2}$$

Given that the cycle ambiguities are fixed incorrectly, the probability that the vertical error exceeds VPL_{H0} (i.e., $P\{|\hat{x}_v - x_v| > VPL_{H0} \mid IF\}$) can conservatively be assumed to be equal to 1. In addition, since both the correct fix and incorrect fix events are mutually exclusive and exhaustive as defined earlier, $PCF = 1 - PIF$. Using the conservative assumption $P\{|\hat{x}_v - x_v| > VPL_{H0} \mid IF\} = 1.0$ in (2) and substituting the result in (1),

$$\begin{aligned} I_{H0\,req} &= P\{|\hat{x}_v - x_v| > VPL_{H0} \mid CF\}\, PCF \\ &\quad + PIF \\ \Rightarrow P\{|\hat{x}_v - x_v| > VPL_{H0} \mid CF\} &= \frac{I_{H0\,req} - PIF}{1 - PIF} \end{aligned} \tag{3}$$

Under the assumption that the GPS measurement noise is zero-mean Gaussian, the vertical position error, after fixing the cycle ambiguities correctly, can be assumed to be a random variable with a Gaussian distribution of zero mean and standard deviation $\sigma_{v|CF}$. Therefore, the probability multiplier ($K_{VPL|CF}$) is calculated using the inverse of the cumulative normal distribution function of $P\{|\hat{x}_v - x_v| > VPL_{H0} \mid CF\}$. For example, for $I_{H0\,req} = 10^{-7}$, in [6] a reasonable threshold for *PIF* was found to be 10^{-8}, which results in $K_{VPL|CF} = 5.35$. Therefore, VPL_{H0} is calculated by multiplying $\sigma_{v|CF}$ by 5.35. This VPL_{H0} must comply with the maximum tolerable vertical error, which is known as the Vertical Alert Limit (*VAL*).

If a tighter accuracy and integrity requirement exists, as we will examine later in Section V, this method is not sufficient to meet the availability requirements. One venue to improve the VPL_{H0} calculation is the conservative assumption, made after (2), that all incorrect integer candidates will cause the position errors to exceed the alert limits. In response, in the following section we introduce an innovative approach to calculate the integrity risk by evaluating the impact of the incorrect integer candidates, weighted by their probability of occurrence, in the position domain.

III. Position Domain Integrity Risk of Ambiguity Fix

Instead of using the conservative assumption that the vertical error exceeds VPL_{H0} if ambiguities are fixed incorrectly, we consider an integer space that includes all candidate sets of cycle ambiguities that cause the position error to fall inside the alert limit boundaries. This includes all sets, correct or incorrect fixes, as long as they cause the vertical error to be bounded by the alert limits. In other words, if a candidate set is incorrect but satisfies the position domain alert limit constraints, we recognize that it does not violate integrity.

A. Mathematical Derivation

In this new method, instead of starting from the definition of VPL_{H0} that meets the integrity requirements, we directly calculate the integrity risk (I_{H0}) caused by the cycle ambiguity candidates and compare it to the required fault free integrity risk (I_{H0req}). If the calculated integrity risk (I_{H0}) is less than the required integrity risk then the navigation system is considered available under fault-free conditions. Fault free integrity risk is defined as the probability that the vertical error exceeds *VAL* as shown in (4).

$$I_{H0} = P\{|\hat{x}_v - x_v| > VAL\} \tag{4}$$

Considering the two mutually exclusive and exhaustive events *IF* and *CF* and using the law of total probability as shown in (2), equation (4) can be rewritten as,

$$\begin{aligned} I_{H0} &= P\{|\hat{x}_v - x_v| > VAL \mid CF\}\, PCF \\ &\quad + P\{|\hat{x}_v - x_v| > VAL \mid IF\}\, PIF \end{aligned} \tag{5}$$

The second term in (5) can be expanded to include all events that correspond to incorrectly fixed cycle ambiguity vector candidates (6).

$$\begin{aligned} &P\{|\hat{x}_v - x_v| > VAL \mid IF\}\, PIF \\ &= \sum_{n=1}^{\infty} P\{|\hat{x}_v - x_v| > VAL \mid IF_n\}\, PIF_n \end{aligned} \tag{6}$$

where,

- IF_n: event corresponding to the n^{th} incorrectly fixed cycle ambiguity vector
- PIF_n: probability of occurrence of the n^{th} incorrect fix.

Since it is impractical to calculate the series with an infinite number of incorrect cycle ambiguity vector candidates, the series is broken into two subseries: one represents what will later be the candidates of interest (n = 1→ k) and another which includes all remaining candidates (n = k + 1→ ∞) as shown in (7).

$$\begin{aligned} &P\{|\hat{x}_v - x_v| > VAL \mid IF\}\, PIF \\ &= \sum_{n=1}^{k} P\{|\hat{x}_v - x_v| > VAL \mid IF_n\}\, PIF_n \\ &+ \sum_{n=k+1}^{\infty} P\{|\hat{x}_v - x_v| > VAL \mid IF_n\}\, PIF_n \end{aligned} \tag{7}$$

At this stage, the conservative assumption $P\{|\hat{x}_v - x_v| > VAL \mid IF_n\} = 1.0$ for n = k + 1→∞ is used. In addition, since the correct fix event and all incorrect fix events (including the tested candidates) are mutually exclusive and exhaustive, the second term in (7) can be written as

$$\sum_{n=k+1}^{\infty} PIF_n = 1 - PCF - \sum_{n=1}^{k} PIF_n \tag{8}$$

Substituting (8) into (7) and using the result in (5) yields

$$\begin{aligned} &I_{H0} \\ &= P\{|\hat{x}_v - x_v| > VAL \mid CF\}\, PCF \\ &+ \sum_{n=1}^{k} P\{|\hat{x}_v - x_v| > VAL \mid IF_n\}\, PIF_n + 1 - PCF \\ &- \sum_{n=1}^{k} PIF_n \end{aligned} \tag{9}$$

Rearranging (9), we obtain

$$\begin{aligned} &I_{H0} \\ &= 1 - (1 - P\{|\hat{x}_v - x_v| > VAL \mid CF\})\, PCF \\ &- \sum_{n=1}^{k} \big((1 - P\{|\hat{x}_v - x_v| > VAL \mid IF_n\})\, PIF_n\big) \end{aligned} \tag{10}$$

Equation (10) is a closed-form expression in which the effect of a set of ambiguity candidates on the position domain integrity risk is explicitly defined. It is noteworthy that if the series term in (10) (which represents the incorrect fix candidates to be tested) is neglected, the remaining expression yields

$$\begin{aligned} I_{H0} &= 1 - (1 - P\{|\hat{x}_v - x_v| > VAL \mid CF\})\, PCF \\ &= PIF + P\{|\hat{x}_v - x_v| > VAL \mid CF\} PCF \end{aligned} \tag{11}$$

which is equivalent to (3) with VAL and I_{H0} replacing VPL_{H0} and I_{H0req}, respectively. As long as the sum of the series in (10) is greater than zero, the integrity risk computed using (10) will be lower than that given by (11). Therefore, equation (10) provides a tighter bound on integrity risk than the conventional method expressed in (11). This in turn will result in improved navigation availability as we will see later in Sections III-E and V.

However, to compute the integrity risk using (10), the terms PCF, PIF_n, $P\{|\hat{x}_v - x_v| > VAL \mid CF\}$, and $P\{|\hat{x}_v - x_v| > VAL \mid IF_n\}$ must be evaluated. For compactness of notation, the latter two terms will be referred to as P_{VALCF} and P_{VALIF}, respectively. In the following sections, the procedure to compute PCF, PIF, P_{VALCF}, and P_{VALIF} is described. In addition, an efficient method to construct cycle ambiguity candidates is developed.

B. *Evaluating PCF and PIF*

Calculating the probability that a certain set of ambiguities is the correct one depends on the fixing method utilized. In this work, the bootstrap method [9] is used for cycle resolution because it provides a closed form *a priori* probability mass function on integer estimation error. The bootstrap rounding method fixes ambiguities sequentially and provides a measure of the PCF at each step of the fixing process. The sequential adjustment is performed according to the cycle ambiguity conditional variances with the ambiguity having the least variance being fixed first. The i^{th} conditional variance ($\sigma^2_{i/I}$), defined as the variance of the ambiguity i conditioned on the previous ambiguities in the set I={1,2,…,i-1} being fixed, is the (i,i) element of the diagonal matrix **D** resulting from the $\mathbf{LDL}^T$ decomposition of the estimated cycle ambiguity covariance matrix. At each step in this sequence (the m^{th} step for example), the probability that the bootstrapped integer estimate (**ă**) (which is an m×1 vector) being any arbitrary integer candidate (**z**) given that the correct fixed ambiguity is (**a**) is given by [9],

$$P(\breve{\mathbf{a}} = \mathbf{z}) = \prod_{i=1}^{m} \left[\Phi\left(\frac{\left(1 - 2\,\mathbf{l}_i^T(\mathbf{a} - \mathbf{z})\right)}{2\sigma_{\frac{i}{I}}}\right) + \Phi\left(\frac{\left(1 + 2\,\mathbf{l}_i^T(\mathbf{a} - \mathbf{z})\right)}{2\sigma_{i/I}}\right) - 1 \right] \tag{12}$$

where,

$\mathbf{l}_i$: the i^{th} column vector of the unit lower triangular matrix ($\mathbf{L}^{-T}$) resulting from $\mathbf{LDL}^T$ decomposition of the ambiguity covariance matrix

m: the number of cycle ambiguities fixed $m \le n$

$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}v^2\right) dv$

Therefore, if we set **z** equal to **a** (the correct cycle ambiguity vector), Equation (12) produces

$$P(\breve{\mathbf{a}} = \mathbf{a}) = \prod_{i=1}^{m} 2\Phi\left(\frac{1}{2\sigma_{i/I}}\right) - 1 \tag{13}$$

which can be used to calculate *PCF*. For *PIF*, the value of (**a** - **z**) can be computed for the incorrect fix candidate of interest. For example, if a one cycle error on the first ambiguity is to be used as the first incorrect fix event (IF_1), then the vector $[1\ 0 \ldots 0]^T$ is used for (**a** - **z**).

C. Evaluating P_{VALCF} and P_{VALIF}

Before estimating the probability of the vertical error exceeding *VAL*, for simplification purposes and without the loss of generality, the conditional events *CF* and IF_n are replaced by a general event *B*. Later on, when we derive a methodology for estimating this probability, we will tackle the difference between the *CF* and *IF* events and discuss the specific approach for each one.

By expanding the absolute value inside the conditional probability term of (10), the probability of the vertical error exceeding *VAL* given that event *B* took place can be written as:

$$P\{|\hat{x}_v - x_v| > VAL \mid B\} = P\{(\hat{x}_v - x_v) < -VAL \mid B\} + P\{(\hat{x}_v - x_v) > VAL \mid B\} \tag{14}$$

Since the position vector is linearly estimated using the GPS measurements (which are assumed to have Gaussian error distributions), the distribution of the residual error would also be Gaussian with a standard deviation $\sigma_{v|B}$. The mean of this distribution will depend on the event *B* and hence is referred to as μ_B. Knowing the mean (μ_B) and standard deviation ($\sigma_{v|B}$), the probability of the first and second terms of (14) can be calculated using the normal cumulative distribution function at the limits -*VAL* and *VAL*, respectively.

Returning to the *CF* and IF_n events, the standard deviation of the vertical position error is not affected by an incorrect fix. Therefore, $\sigma_{v|B} = \sigma_{v|CF}$ for both the *CF* and IF_n events. The only difference between these events is the mean of the normal distribution describing these quantities. Since the position vector is estimated using unbiased estimators (usually least squares or Kalman filter estimation are used), the mean is zero for the *CF* event ($\mu_{CF} = 0$). In the case of IF_n events, the incorrect integer candidate will induce a bias in the position vector estimate. For example, an incorrect fix of one cycle on the first integer would cause a bias in the position vector estimate that can be modeled as measurement error by one cycle in magnitude in the direction of the line of sight vector of that satellite. Therefore, knowing the satellite geometry matrix (**G**), the bias $\mathbf{b}_n$ induced in the relative position vector caused by the n^{th} (**a** - **z**) incorrect candidate can be modeled as,

$$\lambda\,(\mathbf{a} - \mathbf{z})_n = \mathbf{G}\,\mathbf{b}_n \tag{15}$$

where, λ is the carrier wavelength and the bias $\mathbf{b}_n$ can be estimated as follows:

$$\mathbf{b}_n = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\,\lambda\,(\mathbf{a} - \mathbf{z})_n \tag{16}$$

P_{VALIF} can be then calculated using a normal cumulative distribution function with standard deviation $\sigma = \sigma_v$ and a mean ($\mu_{IFn} = \mathbf{b}_n$) where $\mathbf{b}_n$ is calculated using (16).

If requirements other than *VAL*, such as *LAL*, vertical accuracy (Acc_v) or lateral accuracy (Acc_L) exist, Equation (10) can be altered accordingly. Equations (17), (18) and (19) show the corresponding form of (10) for *LAL*, vertical and lateral accuracy requirements, respectively.

$$I_{H0LAL} = 1 - (1 - P\{|\hat{x}_L - x_L| > LAL \mid CF\})\,PCF - \sum_{n=1}^{k}\big((1 - P\{|\hat{x}_L - x_L| > LAL \mid IF_n\})\,PIF_n\big) \tag{17}$$

$$P_{accV} = 1 - (1 - P\{|\hat{x}_v - x_v| > Acc_v \mid CF\})\,PCF - \sum_{n=1}^{k}\big((1 - P\{|\hat{x}_v - x_v| > Acc_v \mid IF_n\})\,PIF_n\big) \tag{18}$$

$$P_{accL} = 1 - (1 - P\{|\hat{x}_L - x_L| > Acc_L \mid CF\})\,P\{CF\} - \sum_{n=1}^{k}\big((1 - P\{|\hat{x}_L - x_L| > Acc_L \mid IF_n\})\,PIF_n\big) \tag{19}$$

where,

I_{H0LAL}: lateral fault free integrity risk

Acc_v: vertical accuracy requirement

P_{accV}: probability that the vertical position error exceeds Acc_v

Acc_L: lateral accuracy requirement

P_{accL}: probability that the lateral position error exceeds Acc_L

Special care should be taken if any lateral requirements exist. In nominal GPS applications the vertical accuracy is usually worse than the lateral accuracy because of the geometry of the satellites (satellites are visible above the horizon only). However in the case of incorrect fixing, biases might exist at different magnitudes and in different directions. Under incorrect fix conditions, it is quite possible to find a few cases under which the vertical requirements are met while the lateral ones are not.

In most navigation applications, the lateral requirements are defined perpendicular to the flight path. Since the geometry of the satellites generally produces different errors in the North and East directions, lateral position error depends on the flight path azimuth. To be conservative in calculating availability, the azimuth that produces the worst accuracy is used. In the correct fix (*CF*) case, the worst case lateral error variance is simply the maximum eigenvalue of the (2×2) horizontal position error covariance matrix. However, when biases of different magnitudes and directions are introduced to the North and East directions (as in the case of incorrect fix), this is not necessarily the case. An alternative method that shows how to find the worst heading for lateral requirements and how to calculate the associated mean (μ_{IF}) and standard deviation (σ) is detailed in Appendix A.

D. Generating Cycle Ambiguity Candidates

In principle, cycle ambiguity candidates can be constructed by directly considering all possible combinations of cycle errors for all ambiguities. As discussed in Section III-A, theoretically, if the series term $\sum_{n=1}^{k}\left((1-P\{|\hat{x}_v - x_v| > VAL \mid IF_n\})\, PIF_n\right)$ in (10) is negligible, then we expect no performance improvement. This series will be close to zero under two scenarios: if P_{VALIF} is large (close to 1.0), or if *PIF* is small (close to zero). P_{VALIF} will be large if the incorrect fix ambiguity candidates introduce large biases (μ_{IF}) in the position domain. On the other hand, from (12) it can be noticed that the larger (**a** – **z**) is, which is the case when the incorrectly fixed ambiguity **z** is farther from the correct one **a** (the one with maximum probability of correct fix), the smaller *PIF* becomes. Therefore, adding more candidates and increasing *k* will not change the series outcome significantly. Therefore, it is not necessary to search the whole integer space (k → ∞) for incorrect fix candidates. It is adequate to use only the ones surrounding the correct fix (**a**) that lead to near convergence of the series in (10).

The following procedure is followed to construct cycle ambiguity candidates:

1. Define a candidate matrix (**C**) as the matrix whose columns represent all the vectors (**a** – **z**) for all incorrectly fixed ambiguity candidates. Since one integer is fixed first, **C** is initialized with a row vector of integer numbers representing (**a** – **z**) candidates for the first ambiguity. Therefore, if *d* is used as an arbitrary integer bound on (**a** – **z**), then $\mathbf{C}_1 = [-d, -d+1, \ldots, -1, 1, \ldots, d-1, d]$. This corresponds to the first step in the bootstrap sequential fixing (fixing one ambiguity only).
2. For each step in the sequential fixing, other than the initial one, update $\mathbf{C}_i$ to include all new combinations for the newly fixed ambiguity. This can be accomplished, for $2 \le i \le n$, by
$$\mathbf{C}_i = \begin{bmatrix} \mathbf{C}_{i-1} & \cdots & \mathbf{C}_{i-1} & \cdots & \mathbf{C}_{i-1} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ -\mathbf{d} & \cdots & \mathbf{0} & \cdots & \mathbf{d} & -\mathbf{d} & \cdots & -\mathbf{1} & \mathbf{1} & \cdots & \mathbf{d} \end{bmatrix}$$
where **d** is a row vector with all of its elements equal to *d*.
3. Calculate *PIF* for each (**a** - **z**) candidate (each column of the **C** matrix).
4. Only relevant candidates that are significant in the calculation of the series in (10) are kept in **C**. Therefore, if PIF for the k^{th} candidate (k^{th} column of **C**) is less than a threshold (for example $0.01 \times I_{H0\,req}$), then the k^{th} column in **C** is eliminated.
5. The resultant **C** is the matrix that includes all ambiguity candidates that are relevant in (10) and will be used in the calculation of I_{H0}.
6. Steps 2 – 5 are repeated for all ambiguities necessary to be fixed to meet the requirements.

Although this procedure is general, it requires a significant amount of memory and is computationally inefficient. However, its efficiency can be greatly enhanced and computation time can be considerably reduced if the Least-square AMBiguity Decorrelation Adjustment (LAMBDA) method is used in addition to the bootstrapping. (More about the LAMBDA decorrelation method for fixing cycle ambiguities can be found in [10].) Because of the decorrelation of ambiguities using LAMBDA, only combinations that include one-cycle-off the correct one are sufficient (as we will see shortly). However, remember that when using LAMBDA the (**a** - **z**) candidates are in the Z-decorrelated space. Therefore, when estimating the bias ($\mathbf{b}_n$), (16) must be replaced by

$$\mathbf{b}_n = (\mathbf{G}^T\mathbf{G})^{-1}\mathbf{G}^T\mathbf{Z}^{-1}\,\lambda\,(\mathbf{a}-\mathbf{z})_n \qquad (20)$$

where **Z** is the integer transformation matrix that maps the cycle ambiguity covariance to the decorrelated space. In addition, the $\mathbf{LDL}^T$ decomposition is performed on the decorrelated ambiguity matrix resulting from the LAMBDA routine.

E. Performance Enhancement Using the Position Domain Integrity Risk Method

In order to demonstrate the gain using the position domain integrity risk method (which we refer to as the '*new method*' for short) over the conventional method explained in Section II, a simple example for a single measurement epoch is used. In this example, a single satellite geometry, which is purposefully chosen to demonstrate the gain using the new method, is used to provide the float cycle ambiguity estimate and the corresponding covariance. First we will illustrate the performance of the conventional method for this geometry. After estimating the float ambiguities, LAMBDA is used to decorrelate the estimated cycle ambiguity covariance matrix and the bootstrap method is used to sequentially fix the ambiguities one at a time. At each step in this sequential fixing process, *PCF* using (13) and the resultant vertical position accuracy (σ_v) are recorded. In Fig. 1, the vertical accuracy and *PIF* (where *PIF* = 1 – *PCF*) are illustrated versus the number of fixed ambiguities during the sequential fixing. In each of these plots the corresponding thresholds which are derived from certain assumed requirements are also shown as dashed lines. For example, in the σ_v plot the dashed line represents the VAL requirement (calculated by dividing the *VAL* (for example 1.1 m is used here) by $K_{VPL|CF} = 5.35$ as shown in Section II). For the PIF plot, the dashed line corresponds to the 10^{-8} PIF threshold that is derived from the integrity requirement as explained in Section II. From Fig. 1, it is obvious that although at least four cycle ambiguities must be fixed to meet the *VAL* requirements, fixing even the first cycle ambiguity violates integrity (shown in the PIF plot). Since both the *VAL* and integrity requirements cannot be met simultaneously, this geometry is considered unavailable if the conventional method in Section II is used.

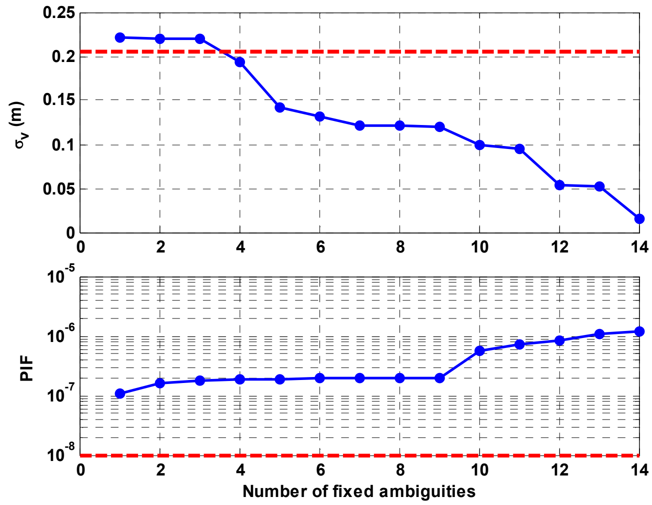

Fig. 1. Vertical Accuracy and PIF Versus the Number of Fixed Ambiguities Using the Conventional Method

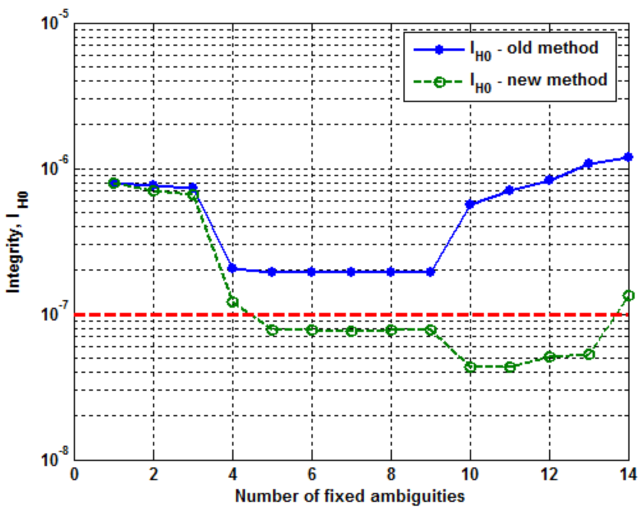

Fig. 2. Integrity as a Function of the Number of Fixed Ambiguities for both the Conventional Method and the New Method

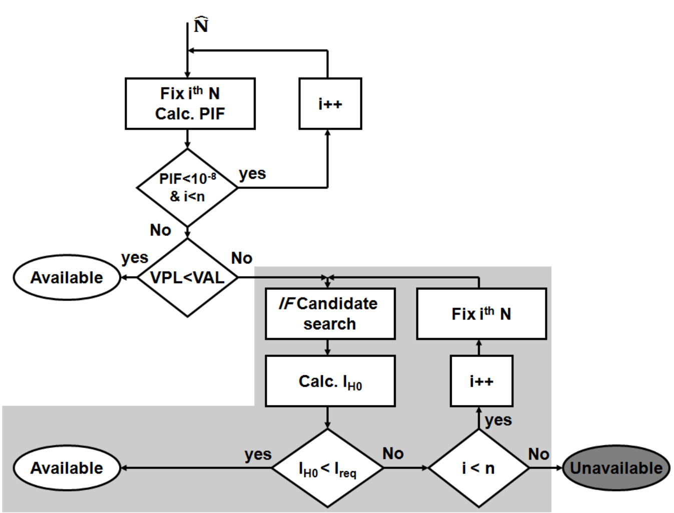

Fig. 3. Schematic Diagram Showing the Structure of the Partial Fixing Algorithm Using the Position Domain Integrity of Ambiguity Fix Method

The results shown in Fig. 1 can be combined in one plot (solid curve in Fig. 2) that shows integrity I_{H0} calculated using (11) (without the series term) versus the number of fixed ambiguities. If I_{H0} exceeds the integrity risk requirement (for example, I_{H0req} = 10^{-7}), the satellite geometry is considered unavailable. Therefore, the solid curve in Fig. 2 confirms that I_{H0} using the conventional method (11) does not meet the integrity risk requirement for this geometry for any number of fixed ambiguities. However, using the new method in (10), the integrity risk requirement is met if more than four ambiguities are fixed. The only exception is fixing all cycle ambiguities because at that point, I_{H0} without the series term (10) reached a limit where it could not be reduced enough to meet the 10^{-7} requirement using the series term. Using LAMBDA, 108 combinations of one-cycle-off candidates were adequate to generate the results shown in Fig. 2. On the other hand, if LAMBDA is not used, incorrect fixes of up to four cycles are necessary, resulting in 1792 candidates.

Fig. 2 illustrates that if an all-fixed solution is implemented, the geometry will be considered unavailable because I_{H0} exceeds the fault free integrity risk requirement in that case. On the other hand, this geometry can be considered available if we fix fewer cycle ambiguities (five for example). Therefore, a partial-fix implementation is recommended. A practical method to choose the number of fixed ambiguities that meets the integrity requirement will be discussed in the next section.

IV. Partial Fixing Using Position Domain Integrity Risk Method

In this section, a procedure to determine the navigation system availability using the position domain integrity risk method with partial fixing is described. In the previous section, it was concluded that a partially fixed solution is necessary to improve the availability using the proposed algorithm.

Fig. 3 shows a flow chart that describes how the final partial fixing algorithm using the new method is implemented in a navigation architecture. After estimating the float ambiguities, the float estimates ($\hat{\mathbf{N}}$ in Fig. 3) are input to the LAMBDA bootstrap fixing algorithm. These ambiguities are fixed sequentially until the *PIF* exceeds the threshold (1×10^{-8}). The remaining linear combinations are left floating. At this point, if the resultant VPL_{H0} is less than *VAL* then this geometry is available and therefore it is unnecessary to go to the second loop, which contains the position domain integrity calculation. Although this step might seem unnecessary using the proposed new method of calculating integrity, it helps reduce the computation time because choosing the incorrect fix candidates is relatively slow and can be avoided for the available cases using the conventional method. If VPL_{H0} is greater than *VAL* the incorrect fix candidates are then constructed for the set of fixed ambiguities up to this point. Using these candidates, I_{H0} using (10) is calculated and compared to the integrity risk requirement I_{H0req}. If I_{H0} is less than I_{H0req}, the geometry is considered available. Otherwise, the next ambiguity is fixed. This process is repeated until an available solution is found or all ambiguities are fixed and I_{H0} is still larger than I_{H0eq}. At this point, the geometry is said to be unavailable.

So far, the performance enhancement using the new method has been demonstrated based on a single geometry only. In order to investigate the potential for performance benefit more thoroughly, an availability analysis for an example shipboard aircraft landing application is conducted using both the conventional and new methods.

V. Availability Performance

A. Navigation Algorithm

Because of the mobility of the reference station in a shipboard-relative landing application, higher levels of accuracy are required than for similar precision approach applications at land-based airfields. In addition, to ensure safety and operational usefulness, the navigation architecture must provide high levels of integrity and availability. Because of the highly stringent requirements, the navigation system is based on CPDGPS positioning. However in order to benefit from the high precision of CPDGPS the correct resolution of cycles must be ensured. A number of methods have been used to obtain the cycle resolution. Satellite motion can provide the observability of the cycle ambiguities [11]. Unfortunately the rate of satellite motion is relatively slow in comparison with the time scales of the analyzed aviation application. However, carrier measurements at the two frequencies (L1 and L2) can be combined to create a beat frequency measurement with a wavelength of 86cm, generally known as the widelane observable. Because of the longer wavelength, the widelane cycle ambiguities are easier to identify using code phase measurements.

Heo, et al. have proposed a GPS navigation algorithm for autonomous shipboard landing applications where geometry free/divergence free code-carrier filtering is performed continuously for visible satellites on both the aircraft and the ship until the aircraft is close to the ship [4] [7]. Geometry-free filtering [12], by definition, does not depend on the geometry of the satellites or the user location and eliminates most of the nuisance terms and errors. A geometry free measurement of the widelane cycle ambiguity can be formed by subtracting the narrowlane pseudorange from the widelane carrier (which leads to eliminating the geometry-dependable and the nuisance terms [11, 12]). A drawback of the geometry free measurement is the presence of higher noise with respect to the L1 and L2 carrier phase measurements. This can be overcome by filtering the geometry free measurement over time preceding the final approach. In order to model multipath colored noise in the geometry free measurements, a first order Gauss-Markov measurement error model with a time constant of one minute for ship and 30 seconds for aircraft is assumed. The outputs of the filtering process are the floating widelane cycle ambiguity estimates. When the aircraft is close to the ship, L1/L2 cycle ambiguity estimates can be extracted with the aid of the geometric redundancy [3, 4]. Next, the cycle ambiguities are fixed using the LAMBDA bootstrap method [9]. For the conventional method, the bootstrap rounding process is performed for those ambiguities that can be fixed with a *PIF* that is lower than the threshold (10^{-8}). The remaining ambiguities remain floating. With the ambiguities resolved, the standard deviation of the position estimate error (σ_v) is used to calculate VPL_{H0}. To account for the GPS satellite geometry change, availability analysis is performed by simulating 1440 satellite geometries (one geometry per minute during the day). Availability is then calculated as the percentage of time (geometries) for which VPL_{H0} is less than *VAL*. For the new method, the position domain integrity of cycle resolution (I_{H0}) is calculated using (10). If I_{H0} is less than the requirements ($I_{H0\,req}$) the geometry is considered available.

B. Availability Results

In this section, the performance of the position domain integrity risk method with partial fixing (Fig. 3) is used and compared to the conventional method (Section II). The requirements and simulation parameters that are used in this work are based on these given in [4] and [7]. Accuracy requirement of 30 cm with P_{acc} = 95% is used to demonstrate the effectiveness of the new proposed method under tighter accuracy requirement than the ones provided by integrity. In this simulation, the standard deviation of the carrier phase (single difference) measurement noise ($\sigma_{\Delta\phi}$) is assumed to be 1.0 cm. In addition, different values for the standard deviation of the pseudorange (single difference) measurement noise ($\sigma_{\Delta\phi PR}$) ranging from 20 cm to 70 cm are used. The single difference standard deviations $\sigma_{\Delta PR}$ and $\sigma_{\Delta\phi}$ are related to the raw values (σ_{PR} and $\sigma_{\Delta\phi}$) by a scaling factor of $\sqrt{2}$. In this analysis, a maximum prefiltering period of 30 minutes is used to generate floating estimates of the widelane cycle ambiguities. In other words, if the satellite has been visible for a period of time longer than the maximum prefiltering time, the prefiltering time is set to the maximum prefiltering time. The rest of the simulation parameters are summarized in Table I.

Using the parameters detailed in Table I, a covariance analysis simulation was performed and availability was calculated. Fig. 4 shows the availability results for both methods using different $\sigma_{\Delta PR}$ values ranging from 20 cm to 70 cm. It can be noticed that the new method outperforms the conventional method by a range of approximately 1% at $\sigma_{\Delta PR}$ = 20 cm to as much as 40% at $\sigma_{\Delta PR}$ = 70 cm. Furthermore, in the case that the availability requirement is 99%, a navigation algorithm with the conventional method can only be used if $\sigma_{\Delta PR}$ = 20 cm. On the other hand, if the new method is used, $\sigma_{\Delta PR}$ of up to 55 cm can be tolerated. However, the results shown in Fig. 4 are for an accuracy requirement of 30 cm. Fig. 5 shows the efficiency of the new method compared to the conventional one under different accuracy requirements at $\sigma_{\Delta PR}$ = 50 cm. Again, the new method outperforms the conventional method and provides up to 78% increase in availability at 15 cm accuracy compared to the conventional one.

TABLE I
SIMULATION PARAMETERS

Parameter	Value
$I_{H0\,req}$	10^{-7}
VAL	1.1 m
LAL	1.1 m
PIF threshold	10^{-8}
Satellite constellation	24 satellite (do229d)
Location	Honolulu (22°N and 158°W)
Prefiltering time	30 minutes
Ship multipath time constant	1 minute
Aircraft multipath time constant	30 seconds

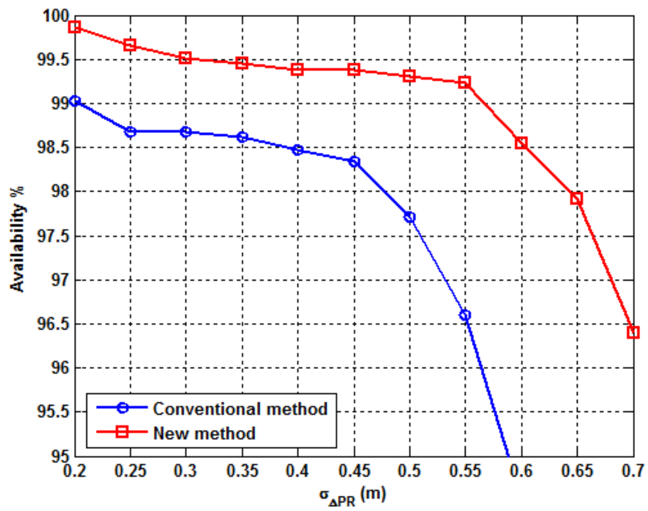

Fig. 4. Availability Results for Different $\sigma_{\Delta PR}$ Values Using both the Conventional Method and New Method

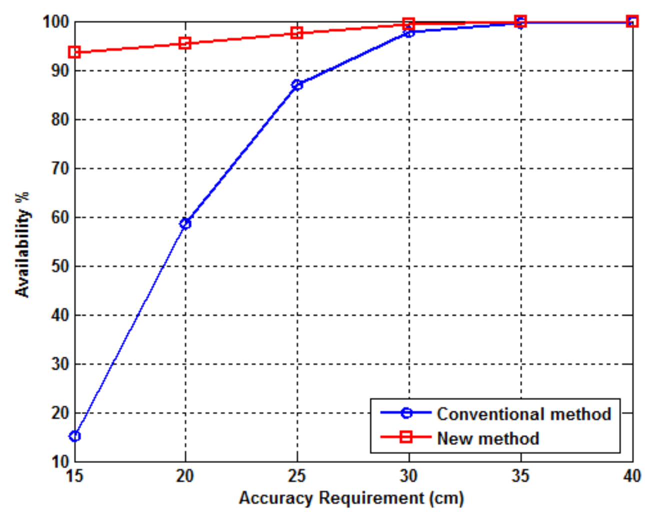

Fig. 5. Availability Results for Different Accuracy Requirements Using both the Conventional Method and New Method at $\sigma_{\Delta PR}$ = 50 cm

In summary, the position domain integrity risk method can be greatly beneficial in navigation applications with high accuracy, stringent integrity and tight availability requirements. Although the numerical calculations in this work have been implemented using the LAMBDA bootstrap method, LAMBDA was only used to increase the computational efficiency. Furthermore, the integrity risk formulas derived are applicable to other cycle resolution algorithms as long as they are capable of quantifying the probability of fixing on any given integer candidate set.

VI. CONCLUSIONS

An innovative method was developed to calculate the fault free integrity risk for cycle resolution in carrier phase navigation algorithms by evaluating the impact of incorrect fixes in the position domain. In addition, a mechanism to implement this method with a partially fixed solution in a navigation system was described. The improvement in navigation availability using this partial fixing method was quantified for an example shipboard landing relative navigation application. The availability analysis demonstrated that this new method provides considerable availability enhancement relative to existing methods.

APPENDIX A

In calculating integrity and availability, worst case scenarios are usually pursued. Using the vertical error component σ_v of the estimated position vector, P_{VAL} can be directly calculated. In contrast, the lateral direction for the *LAL* is defined as the direction perpendicular to the direction of motion of the vehicle. The relative position vector estimation provides a covariance matrix (**P**) for the north and east directions, which can be represented as a covariance ellipse. If this covariance ellipse is centered at the origin (zero mean error), the direction of the major axis of this ellipse represents the direction (heading) corresponding to the maximum sigma lateral (σ_L). Therefore, in the case of a correct fix, the worst P_{LALCF} is based on σ_L which equals to the maximum eigenvalue of the 2×2 covariance matrix of the position vector estimate error. However, in the incorrect fix case, biases in the north and east directions (μ_x and μ_y) are introduced. Therefore, the covariance ellipse is centered at (μ_x,μ_y) instead of the origin (Fig. 6). As a result, the direction of the major axis will not necessarily correspond to the worst P_{LALIF}. In this work, all different angles (θ) from 1 to 360 degrees, in one degree increments, are tested and P_{LAL} are calculated and the maximum corresponds to P_{LALIF}.

In order to calculate the mean (μ) and standard deviation (σ) that are used in calculating P_{LALIF}, a coordinate rotation by angle θ is performed on the covariance ellipse. This can be performed by first computing the position mean magnitude (μ_p) and the direction of the mean (α) as follows:

$$\mu_p = \sqrt{\mu_x^2 + \mu_y^2}$$
$$\alpha = \tan^{-1}\left(\frac{\mu_y}{\mu_x}\right) \qquad (21)$$

Therefore, the projection of the mean on the new direction, which is the mean that is used in calculating *LAL*, is:

$$\mu = \mu_p \cos(\theta - \alpha) \quad (22)$$

If **P** is the 2×2 covariance of the estimated position corresponding to the north and east directions, the projection of the covariance to calculate sigma (σ) can be calculated using:

$$\sigma = [\cos(\theta) \quad \sin(\theta)]\, \mathbf{P} \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} \quad (23)$$

Finally, P_{LALIF} which is defined as $P\{|\hat{x}_v - x_v| > LAL \mid IF_n\}$ is calculated using the normal cumulative distribution function with a mean μ and standard deviation σ at $x = LAL$.

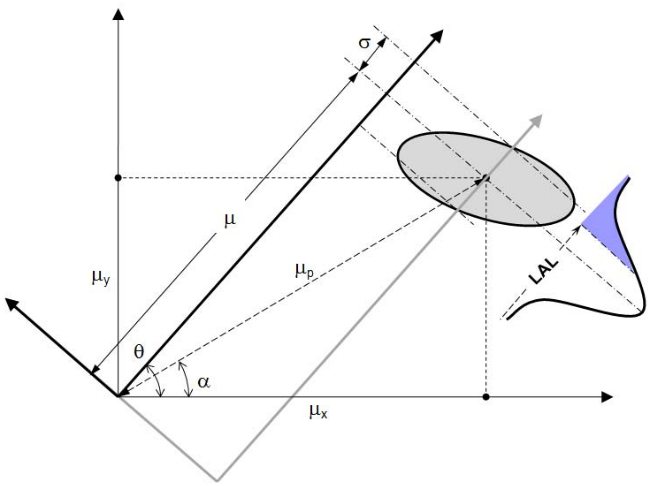

Fig. 6. Calculating the Mean and Standard Deviation of the Projection of the Covariance Ellipse with an Angle θ

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