Carrier Phase Relative RAIM Algorithms and Protection Level Derivation

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Abstract

The concept of Relative Receiver Autonomous Integrity Monitoring (RRAIM) using time differential carrier phase measurements is investigated in this paper. The precision of carrier phase measurements allows for mitigation of integrity hazards by implementing RRAIM monitors with tight thresholds without significantly affecting continuity. In order to avoid the need for cycle ambiguity resolution, time differences in carrier phase measurements are used as the basis for detection. In this work, we examine RRAIM within the context of the GNSS Evolutionary Architecture Study (GEAS), which explores potential architectures for aircraft navigation utilizing the satellite signals available in the mid-term future with GPS III. The objectives of the GEAS are focused on system implementations providing worldwide coverage to satisfy LPV-200 operations, and potentially beyond. In this work, we study two different GEAS implementations of RRAIM. General formulas are derived for positioning, fault detection, and protection level generation to meet a given set of integrity and continuity requirements.

Introduction

Carrier phase differential RAIM implementations have been investigated in prior work to help detect specific navigation threats, including ephemeris broadcast anomalies [1], and ionospheric storm fronts [2]. The great precision of carrier phase measurements allowed for tight detection thresholds without significantly affecting continuity. It also provided the sensitivity to detect a much larger range of failure magnitudes than possible using traditional code-based RAIM. The need for cycle ambiguity estimation was eliminated by differencing measurements in time, creating spatial baselines associated with the user's translation over the time-difference interval. We refer globally to these time-differenced carrier phase RAIM implementations as Relative RAIM (RRAIM) functions. The results in [1, 2] showed that many troublesome ionospheric and ephemeris threats could be detected with carrier phase RRAIM implementations, even for applications where positioning was based on code phase [2].

The current process of modernization of Global Navigation Satellite Systems (GNSS) will improve navigation user capabilities in many ways, two of them being particularly relevant to this work: there will be an additional civil signal available, and there will be a larger number of satellites in view at all times. This will translate into more precise ranging (ionospheric delay is eliminated using dual frequency measurements) and greater redundancy. Ranging precision and redundancy are naturally two key elements affecting the efficiency of RAIM implementations.

In parallel, the Federal Aviation Administration (FAA) has organized a GNSS Evolutionary Architecture Study (GEAS) group to explore the different possibilities for aircraft navigation utilizing the new satellite signals available in the mid-term future. The objective of GEAS is to develop new navigation architectures for aviation to provide worldwide coverage for aircraft precision approach, initially LPV-200 operations, with minimal ground infrastructure. In response, the GEAS has focused its investigations on three different architectures, one of which is based on RRAIM. These were first described in [3], and are summarized briefly below.

The first architecture most nearly resembles the existing Wide Area Augmentation System (WAAS), which is already capable of achieving LPV-200 performance, but not globally. The GEAS-equivalent concept is called the GNSS Integrity Channel (GIC) architecture. It is WAAS-like in that it will use sparsely placed differential reference stations to generate ranging corrections and perform integrity monitoring. However, ionospheric corrections are not required because L5 Global Positioning System (GPS) signals are assumed to be available to airborne users. This means that a global network of GIC ground station can be widely spaced, requiring perhaps only 20 stations worldwide. Based on WAAS experience, one challenge for this architecture is that it may be difficult to meet aircraft time-to-alert (TTA) requirements (6 seconds for LPV 200) with an integrated global system.

The second concept is called the Absolute RAIM (ARAIM) architecture. This approach is essentially a traditional RAIM architecture that uses carrier-smoothed code measurements for both positioning and fault detection. In this concept, the integrity burden is placed almost entirely at the aircraft. A simplified version of the GIC fault detection function is needed only to ensure that the prior probability of undetected multiple, simultaneous satellite faults is kept low. The ARAIM implementation uses smoothed code in its RAIM test, and depends heavily on redundancy, so it is very demanding on satellite constellation. Initial results suggest that achieving good availability for worldwide LPV-200 with ARAIM requires constellations of 30 or more satellites [3]. The choice of this architecture would therefore presume a suitably expanded GPS constellation or a combined-constellation GNSS.

The final GEAS concept is called the 'RRAIM' architecture. Like the previous two concepts, this system uses carrier-smoothed code for positioning. However, fault detection is performed using a combination of GIC ground based monitoring and a carrier phase RRAIM function. This architecture represents a practical intermediate solution between the GIC and ARAIM architectures. It relies heavily on integrity monitoring capability of a global WAAS-like GIC, but eliminates the resulting TTA concern by providing integrity for the latest segment of flight with a carrier phase RRAIM function. Preliminary analysis in [3] showed that a RRAIM implementation could potentially enable LPV-200 operations with worldwide coverage using a 27 SV constellation [3].

In the RRAIM navigation architecture, an initial position estimate is obtained using data whose integrity is validated by the GIC. Because there is a delay between the epoch when corrections are computed by the GIC and the moment the user receives them, the aircraft will first use stored carrier-smoothed code measurements, corresponding to the time of generation of the last received GIC correction, to obtain a 'reference' position $\boldsymbol{x_0}$. The integrity of the reference position is therefore ensured by the GIC. A relative vector is then computed using time-differential carrier phase measurements and added to $\boldsymbol{x_0}$ to determine the current position. The integrity of this relative vector is provided by a RRAIM test. The duration of the differential time interval is often referred to as the carrier 'Coasting Time' (CT).

As noted earlier, the principal advantage of RRAIM is that it allows for significant relaxation of the GIC TTA requirement. This is true because the RRAIM function ensures navigation integrity after the latest available GIC correction. In the airborne realization of the concept, there are two interesting implications to consider: the effect of increasing coasting time on differential ranging measurement errors (some error sources will increase as the coasting time gets larger), and the potential advantages of looking for the 'best' reference epoch rather than using the latest available one (reference position error depends heavily on satellite geometry).

The focus of this paper is to provide a detailed mathematical development of the GEAS RRAIM concept and to derive the associated algorithms for positioning, fault detection, and position-domain protection level bounding. Two different realizations of the RRAIM concept are developed: these are a 'range domain' implementation and a 'position domain' implementation, which differ in the way the available measurements at the reference and current positions are merged.

RRAIM Algorithms

In general, before executing the approach or landing operation, the aircraft must evaluate the capabilities of its monitors to detect a significant position error if such error existed. When a position error big enough to be a hazard is not detected by the monitors, the pilot (or autopilot) is said to be using Hazardously Misleading Information (HMI). The possibility of this happening is known as Integrity Risk and its likelihood is expressed as the Probability of Hazardously Misleading Information (P_{HMI}). If P_{HMI} is bigger than a certain required specification (P_{HMIreq}), then the approach cannot be initiated. In this work, the fault detection function is based on the weighted least squares residual. Lee in [4] discusses a solution separation algorithm as an alternative detection approach.

For RRAIM, the necessary condition to start an operation can be written as

$$P\left\{\left(|\,\delta x_p\,| > AL\right)\cap\left(r < T\right)\right\} < P_{HMIreq} \qquad (1)$$

where

- δx_p is the error in the user position estimation ,
- AL is the Alert Limit, which is the minimum position error magnitude considered hazardous,
- r is the residual generated by the RRAIM monitor, and
- T is the monitor threshold.

An equivalent way to implement inequality (1) is to derive a Protection Level (PL) such that

$$P\left\{\left(|\,\delta x_p\,| > PL\right)\cap\left(r < T\right)\right\} = P_{HMIreq}\,, \qquad (2)$$

and verify that

$$PL < AL\,. \qquad (3)$$

Potential contributors to P_{HMI}, or equivalently PL, are fault-free (FF) errors (due for example, to an 'unlucky' combination of the nominal errors from all ranging sources), or it could be caused by a failure. For all cases considered in this paper, it is assumed the GIC provides a failure detection and removal rate that ensures that the probability of multiple simultaneous failed ranging sources is negligible.

The RRAIM test is responsible for the integrity from the last GIC correction time to the current time, which we have defined as the Coasting Time (CT). During the CT interval, two situations (mentioned above) are possible: there is a Fault During Coasting (FDC), and that there is Fault Free Coasting (FFC). These two events are mutually exclusive and exhaustive. Because the random parts of δx_p and r are independent, (2) can be written as:

$$P\left(|\delta x_p| > PL \middle| FFC\right) P\left(r < T \middle| FFC\right) P(FFC) + P\left(|\delta x_p| > PL \middle| FDC\right) P\left(r < T \middle| FDC\right) P(FDC) = P_{HMIreq} \quad (4)$$

Ideally we would want to compute one PL that will satisfy (4). However, this is difficult in practice because it typically requires an iterative process that can be very time consuming. A more conservative but practical approach will be to find two PL values, for each hypothesis separately. For example, the overall risk requirement P_{HMIreq} can be sub-allocated into separate components for the two hypotheses, $P_{HMIreq(FFC)}$ and $P_{HMIreq(FDC)}$, such that $P_{HMIreq} = P_{HMIreq(FFC)} + P_{HMIreq(FDC)}$. Then the protection level under the FFC hypothesis is defined by

$$P\left(|\delta x_p| > PL_{FFC} \middle| FFC\right) P\left(r < T \middle| FFC\right) P(FFC) \approx P\left(|\delta x_p| > PL_{FFC} \middle| FFC\right) = P_{HMIreq(FFC)} \quad (5)$$

where it has been conservatively assumed that $P\left(r < T \middle| FFC\right) P(FFC) \approx 1$. Similarly, for the FDC hypothesis,

$$P\left(|\delta x_p| > PL_{FDC} \middle| FDC\right) P\left(r < T \middle| FDC\right) P(FDC) = P_{HMIreq(FDC)} \quad (6)$$

Inequality (3) can then be re-expressed as:

$$PL = \max(PL_{FFC}, PL_{FDC}) < AL \quad (7)$$

Note that an incorrect allocation of P_{HMIreq} between $P_{HMIreq(FFC)}$ and $P_{HMIreq(FDC)}$ is not an integrity risk, but it might impact the availability of the system by conservatively making one of the two PL s excessively large.

In the following development, we will provide a conservative and practical way to compute the PL. The right hand sides of (5)-(7) are system integrity risk requirements, and the detection threshold T is derived from the system continuity risk requirement. The necessary intermediate steps toward computing the PL are to statistically describe the RRAIM residual r and the position error δx_p.

The RRAIM architecture uses two sets of user-satellite ranges (to be described in detail shortly): z_ϕ, composed of time differential carrier phase measurements, and z_C composed of carrier smoothed code measurements.

Two different RRAIM architecture implementations will be developed. One merges z_ϕ and z_{C0} (z_C at time '0') in the Range Domain (RD) to obtain the user current position and clock bias estimate $\hat{x}$. [Note: throughout the paper, the absence of a time subscript implies 'current time.'] The second implementation uses z_{C0} to obtain an initial estimate $\hat{x}_0$, and then uses z_ϕ to obtain a relative vector estimate $\Delta\hat{x}$ that will be added to $\hat{x}_0$ to obtain $\hat{x}$. In this case the information is combined in the Position Domain (PD).

Range Domain Implementation

For each GNSS space vehicle (SV) the user will have a GIC corrected (carrier smoothed) code measurement at a past epoch '0'. The corrected measurement for SV '*i*' can be described as:

$$z_{C0}^{*i} = l_0^i + \tau_0 + \nu_{C0}^{*i} + b_0^i = e_0^{i\,T}\left(x_{SV0}^i - x_{p0}\right) + \tau_0 + \nu_{C0}^{*i} + b_0^i \quad (8)$$

where:

- l^i is the actual user-SV range,
- τ is the receiver clock bias
- ν_C^{*i} represents the sum of SV clock error ($\nu_{C\tau}^i$) after the GIC corrections have been applied, residual tropospheric delay (ν_{CTropo}^i) after model-based correction, and multipath and receiver noise for carrier smoothed code (ν_{CMP+n}^i),
- e^i is the user-SV line of sight unit vector, and
- x_{SV}^i is the SV position.
- b^i is a bias that is included to account for two things: (1) a failure small enough to go undetected by the GIC, and (2) errors that cannot be easily modeled as Gaussian, and will instead be characterized by bounds on their potential magnitudes.

(Throughout the paper a superscript '*T*' indicates the matrix or vector is transposed.)

Even though b_0^i might contain a small failure, this is a failure from the point of view of the GIC, which is responsible for detecting it. For the RRAIM algorithm, as long as its value is bounded, there is no need to distinguish the source of the bias. In other words, only an error occurring during carrier coasting distinguishes between FFC and FDC events. To account for b_0^i, a bound β^i on the magnitude of b_0^i will be chosen such that the $P\left(|b_0^i| > \beta^i\right)$ for all n SVs is negligible compared with P_{HMIreq}:

$$P_{HMIreq} - \sum_{i=1}^{n} P\left(|b_0^i| > \beta^i\right) \approx P_{HMIreq} \tag{9}$$

The choice of bound β^i depends solely on the integrity monitor capabilities of the GIC.

We will now re-express our measurement in (8) as

$$z_{C0}^i = z_{C0}^{*i} - e_0^{i^T} \hat{x}_{SV0}^i = -e_0^{i^T} x_{p0} + \tau_0 + \nu_{C0}^i + b_0^i \tag{10}$$

where $\hat{x}_{SV0}^i$ is the ephemeris generated (and GIC corrected) position of the satellite. The error in the term $e^{i^T} \hat{x}_{SV}^i$ is

$$\nu_{Ceph}^i = e^{i^T} \delta x_{SV}^i , \tag{11}$$

and it is now included in the ν_C^i term:

$$\nu_{C0}^i = \nu_{CMP+n0}^i + \nu_{C\tau 0}^i + \nu_{CTropo0}^i + \nu_{Ceph0}^i \tag{12}$$

All terms in (12) are the errors after all GIC corrections have been applied.

The user will also have a stored carrier phase measurement from time 0 for any SV i:

$$\begin{aligned} z_{\phi 0}^{**i} &= l_0^i + \tau_0 + N^i + \nu_{\phi 0}^{*i} + f_0^{*i} \\ &= e_0^{i^T}\left(x_{SV0}^i - x_{p0}\right) + \tau_0 + N^i + \nu_{\phi 0}^{*i} + f_0^{*i} \end{aligned} \tag{13}$$

where:

- N^i is the carrier phase cycle ambiguity,
- ν_ϕ^{*i} represents the sum of SV clock error ($\nu_{\phi\tau}^i$) after the GIC corrections have been applied, residual tropospheric delay ($\nu_{\phi Tropo}^i$) after model-based correction, and multipath and receiver noise for the carrier phase measurement ($\nu_{\phi MP+n}^i$), and
- f_0^{*i} is a potential failure affecting the measurement.

As we did for the code measurement, we re-express our carrier phase measurement at 0 as

$$\begin{aligned} z_{\phi 0}^{*\,i} &= z_{\phi 0}^{**i} - e_0^{i^T} \hat{x}_{SV0}^{\;i} \\ &= -e_0^{i^T} x_{p0} + \tau_0 + N^i + \tau_{SV0}^{\;i} + \nu_{\phi 0}^{\;i} + f_0^{*i} \end{aligned} \tag{14}$$

and then do the same for the current time as

$$z_\phi^{*i} = -e^{i^T} x_p + \tau + N^i + \nu_\phi^{\;i} + f^{*i} , \tag{15}$$

where

$$\nu_\phi^i = \nu_{\phi MP+n}^i + \nu_{\phi\tau}^i + \nu_{\phi Tropo}^i + \nu_{\phi eph}^i . \tag{16}$$

It is important to note that the measurement corrections that lead to the residual errors in (12) and (16) are different. For the code measurements (10) at time 0, validated GIC corrections are applied. For the carrier measurements (14) and (15), GIC corrections are not applied (because their integrity is not assured for the current time.) The carrier measurements use only the GPS broadcast ephemeris and clock corrections. However, the same tropospheric model is applied for all measurements (although the correction itself may differ between time 0 and the current time). More details on the error models can be found in [3].

The time-differential carrier phase measurement that will be used in the RRAIM system is

$$z_\phi^i = z_\phi^{*i} - z_{\phi 0}^{*i} = -e^{i^T} x_p + e_0^{i^T} x_{p0} + \Delta\tau + \Delta\nu_\phi^i + f^{\,i} \tag{17}$$

Where

$$\Delta\tau = \tau - \tau_0; \quad \Delta\nu_\phi^i = \nu_\phi^i - \nu_{\phi 0}^i; \quad f^{\,i} = f^{*i} - f_0^{*i} . \tag{18}$$

We can now define our RD basic measurement at the current time from (10) and (17) as:

$$\begin{aligned} z_{RD}^i = z_{C0}^i + z_\phi^i &= -e^{i^T} x_p + \tau + \nu_{C0}^i + \Delta\nu_\phi^i + b_0^i + f^{\,i} \\ &= h^{i^T} x + \nu_{C0}^i + \Delta\nu_\phi^i + b_0^i + f^{\,i} \end{aligned} \tag{19}$$

where

$$x = \begin{bmatrix} x_p & \tau \end{bmatrix}^T ; \qquad h^i = \begin{bmatrix} -e^{i^T} & 1 \end{bmatrix}^T . \tag{20}$$

The state vector estimate will then be obtained from

$$\hat{x}_{RD} = \left(H^T R_{z_{RD}}^{-1} H\right)^{-1} H^T R_{z_{RD}}^{-1} z_{RD} , \tag{21}$$

where

$$H = \begin{bmatrix} h^1 & \cdots & h^n \end{bmatrix}^T, \quad z_{RD} = \begin{bmatrix} z_{RD}^1 & \cdots & z_{RD}^n \end{bmatrix}^T , \tag{22}$$

n is the number of SVs whose signals have been continuously tracked by the user between time 0 and the current time, and $R_{Z_{RD}}$ is the covariance matrix of the Gaussian errors in (19): $v_{C0}^i + \Delta v_\phi^i \ (i = 1, \dots, n)$.

It is worth mentioning that the weighting in (21) might not be strictly optimal, as it is based on only the Gaussian parts of the measurement errors. However, distributions of b_0^i are unlikely to be available, so the Gaussian weighting is used here. Naturally, the effects of b_0^i on position error must be accounted for, and this will be addressed shortly.

The RRAIM residual r will be identical for both the RD and the PD implementations, and, as will become obvious shortly, it is more practical to present it in the PD section that follows.

Position Domain Implementation

From (10) and (20)

$$z_{C0}^i = h_0^{i^T} x_0 + v_{C0}^i + b_0^i \tag{23}$$

and the initial position and clock bias estimate can be obtained as:

$$\hat{x}_0 = \left(H_0^T R_{z_{C0}}^{-1} H_0\right)^{-1} H_0^T R_{z_{C0}}^{-1} z_{C0} \tag{24}$$

where

$$z_{C0} = \begin{bmatrix} z_{C0}^1 & \cdots & z_{C0}^m \end{bmatrix}^T, \tag{25}$$

m is the number of SVs available at time 0, and the rows of H_0 are $h_0^{i^T}$ for each satellite i. $R_{Z_{co}}$ is the covariance matrix of the Gaussian errors in (19): $v_{C0}^i \ (i = 1, \dots, m)$.

From (17), (18) and (20):

$$\begin{aligned} z_\phi^i &= h^{i^T} x - h_0^{i^T} x_0 + \Delta v_\phi^i + f^i \\ &= \Delta h^{i^T} x_0 + h^{i^T} \Delta x + \Delta v_\phi^i + f^i \end{aligned} \tag{26}$$

where

$$\Delta x = x - x_0 \ ; \quad \Delta h^i = h^i - h_0^i . \tag{27}$$

Using the result of (24), we now define the time-differenced carrier phase measurement for the PD implementation:

$$z_{PD\phi}^i = z_\phi^i - \Delta h^{i^T} \hat{x}_0 = h^{i^T} \Delta x + \Delta v_{PD\phi}^i + f^i \tag{28}$$

where the Line Of Sight (LOS) change generated errors in the $\Delta h^{i^T} \hat{x}_0$ term,

$$v_{\Delta LOS}^i = \Delta h^{i^T} \delta x_0 , \tag{29}$$

are included in $\Delta v_{PD\phi}^i$:

$$\Delta v_{PD\phi}^i = \Delta v_\phi^i + v_{\Delta LOS}^i \tag{30}$$

We can now estimate the relative position vector as:

$$\Delta\hat{x} = \left(H^T R_{z_{PD\phi}}^{-1} H\right)^{-1} H^T R_{z_{PD\phi}}^{-1} z_{PD\phi} \tag{31}$$

where

$$z_{PD\phi} = \begin{bmatrix} z_{PD\phi}^1 & \cdots & z_{PD\phi}^n \end{bmatrix}^T, \tag{32}$$

n is the number of SVs continuously tracked between time 0 and the current time, and the rows of H are h^{i^T} for each satellite i. $R_{Z_{PD\phi}}$ is the covariance matrix of the Gaussian errors in (30): $\Delta v_{PD\phi}^i \ (i = 1, \dots, n)$.

Using the results of (24) and (31), we obtain the PD implementation state vector estimate as:

$$\hat{x}_{PD} = \hat{x}_0 + \Delta\hat{x} . \tag{33}$$

Note that $\hat{x}_{PD}$ and $\hat{x}_{RD}$ are two different estimates of the same vector x. The most obvious difference is that in the RD implementation SVs that are not present at the current time are not used at all, while in the PD implementation all SVs present at epoch 0 are used to obtain $\hat{x}_0$.

The RRAIM test statistic (in both implementations) will be computed to detect a potential failure f (see (18) and (28)) occurring during the coasting period:

$$r^2 = \left(z_{PD\phi} - H\Delta\hat{x}\right)^T R_{z_{PD\phi}}^{-1} \left(z_{PD\phi} - H\Delta\hat{x}\right) \tag{34}$$

If all the errors in $z_{PD\phi}$ (i.e., $\Delta v_{PD\phi}^i = \Delta v_\phi^i + v_{\Delta LOS}^i$) were Gaussian, r^2 would be χ^2 distributed with n–4 Degrees Of Freedom (DOF). However, the non-Gaussian term b_0^i, defined in (8), can lead to a non-Gaussian contribution of $v_{\Delta LOS}^i$. For short coasting times (a few minutes), SV geometry change is small and therefore $v_{\Delta LOS}^i$ is also small. Nevertheless, we will assume the actual distribution can be treated as a non-central χ^2 distribution. This will be discussed at greater length later in the paper.

Covariance Matrices

In this section we will explicitly define the elements of the covariance matrices $R_{z_{C0}}$, $R_{z_{RD}}$ and $R_{z_{PD\phi}}$. These are needed for the RD and PD positioning algorithms defined above, as well as for RRAIM residual generation.

To obtain the elements of $R_{z_{C0}}$ from (23) and (12), we assume that all ranging sources and component sources have independent errors:

$$R_{z_{C0}(i,i)} = E\left[{\nu_{C0}^{i}}^{2} \right] = \left(\sigma_{C0}^{i}\right)^2 \quad , \quad R_{z_{C0}(i,j)} = 0 \tag{35}$$

$$\left(\sigma_{C0}^{i}\right)^2 = \left(\sigma_{CMP+n0}^{i}\right)^2 + \left(\sigma_{C\tau 0}^{i}\right)^2 + \left(\sigma_{CTropo0}^{i}\right)^2 + \left(\sigma_{Ceph0}^{i}\right)^2 \tag{36}$$

Representative values for the standard deviations in (36) can be found in [3].

Similarly, the elements of $R_{Z_{RD}}$, from (12), (16), (18) and (19) can be expressed as

$$\begin{aligned} R_{Z_{RD}(i,i)} &= E\left[\left(\nu_{C0}^{i} + \Delta \nu_{\phi}^{i} \right)^2 \right] \\ &= \left(\sigma_{C0}^{i}\right)^2 + \left(\sigma_{\Delta\phi}^{i}\right)^2 - 2E\left[\nu_{CMP+n0}^{i}\nu_{\phi MP+n0}^{i}\right] - 2E\left[\nu_{C\tau 0}^{i}\nu_{\phi\tau 0}^{i}\right] \\ &\quad - 2E\left[\nu_{CTropo0}^{i}\nu_{\phi Tropo0}^{i}\right] - 2E\left[\nu_{Ceph0}^{i}\nu_{\phi eph0}^{i}\right] \end{aligned} \tag{37}$$

where

$$\left(\sigma_{\Delta\phi}^{i}\right)^2 = \left(\sigma_{\Delta\phi MP+n}^{i}\right)^2 + \left(\sigma_{\Delta\phi\tau}^{i}\right)^2 + \left(\sigma_{\Delta\phi Tropo}^{i}\right)^2 + \left(\sigma_{\Delta\phi eph}^{i}\right)^2 \tag{38}$$

Again typical values for the standard deviations in (38) can be found in [3], but it is important to note that in contrast to the values in (36), some of the standard deviations in (38) will be a function of CT.

Equation (37) is a general formula that can be simplified further. Modeling the correlation between each pair of code and carrier measurement errors at time 0 in the last four terms of (37) is complicated, but it is safe to assume each pair is positively correlated. This implies these terms will only reduce the magnitude of $R_{Z_{RD}(i,i)}$. To avoid modeling this correlation, we can conservatively assume all these terms are zero. In addition, the tropospheric error component can be more carefully accounted for in the RD implementation by noting that the same tropospheric model will be used for both code and carrier. Therefore the tropospheric modeling error at epoch 0 will cancel in the RD measurement, leaving only the residual unmodelled tropospheric error at the current time:

$$E\left[\left(\nu_{CTropo0}^{i} + \Delta \nu_{\phi Tropo}^{i}\right)^2\right] = \left(\sigma_{\phi Tropo}^{i}\right)^2 \tag{39}$$

Using equations (36), (37), and (39) we can then write

$$\begin{aligned} R_{Z_{RD}(i,i)} &\approx \left(\sigma_{CMP+n0}^{i}\right)^2 + \left(\sigma_{C\tau 0}^{i}\right)^2 + \left(\sigma_{Ceph0}^{i}\right)^2 + \left(\sigma_{CTropo}^{i}\right)^2 \\ &\quad + \left(\sigma_{\Delta\phi MP+n}^{i}\right)^2 + \left(\sigma_{\Delta\phi\tau}^{i}\right)^2 + \left(\sigma_{\Delta\phi eph}^{i}\right)^2 \end{aligned} \tag{40}$$

and

$$R_{Z_{RD}(i,j)} = 0 \tag{41}$$

For the position domain implementation, from (24), (29) and (30):

$$\begin{aligned} R_{PD\phi(i,i)} &= E\left[\left(\Delta \nu_{\phi}^{i} + \nu_{\Delta LOS}^{i}\right)^2\right] \\ &= \left(\sigma_{\Delta\phi}^{i}\right)^2 + E\left[\left(\nu_{\Delta LOSg}^{i}\right)^2\right] = \left(\sigma_{\Delta\phi}^{i}\right)^2 + \Delta {h^{i}}^{T} S_{R_{Z_{C0}}} \Delta h^{i} \end{aligned} \tag{42}$$

where we have defined (in general, and for the remainder of the paper) for a certain time t and weighting matrix R_{yt}^{-1}:

$$S_{R_{yt}} = \left(H_t^T R_{yt}^{-1} H_t\right)^{-1}, \tag{43}$$

and have also introduced new notation by adding a letter 'g' in the $\nu_{\Delta LOSg}^{i}$ subscript, meaning we are only considering the Gaussian components of the term.

For the non-diagonal elements $R_{PD\phi(i,j)}$ we can assume error sources for different SVs are independent, except for the $\nu_{\Delta LOS}$ term, as the error in $\hat{x}_0$ affects all SVs. Taking the Gaussian component of $\nu_{\Delta LOS}$:

$$R_{Z_{PD\phi}(i,j)} = \Delta {h^{i}}^{T} S_{R_{Z_{C0}}} \Delta h^{j} \tag{44}$$

Estimation Errors

For the range domain implementation, from (21):

$$\delta x_{RD} = S_{R_{Z_{RD}}} H^T R_{Z_{RD}}^{-1} \delta z_{RD} \tag{45}$$

Now defining the vectors

$$\begin{aligned} b_V &= \left[b^1 \cdots b^n\right]^T , \quad \nu_{C0} = \left[\nu_{C0}^{1} \cdots \nu_{C0}^{n}\right]^T , \\ \Delta \nu_{\phi} &= \left[\Delta \nu_{\phi}^{1} \cdots \Delta \nu_{\phi}^{n}\right]^T , \quad f = \left[f^1 \cdots f^n\right]^T \end{aligned} \tag{46}$$

and using (19), equation (45) can be rewritten as:

$$\begin{aligned} \delta x_{RD} &= S_{R_{Z_{RD}}} H^T R_{Z_{RD}}^{-1} \left[\nu_{C0} + \Delta \nu_{\phi} + b_V + f\right] \\ &= A\left[\nu_{C0} + \Delta \nu_{\phi} + b_V + f\right] \end{aligned} \tag{47}$$

(Recall that according to our assumptions, only one of the elements in f is non zero.)

For the PD implementation from (23), (24), (28), (29), (31) and (33):

$$\begin{aligned}\delta x_{PD} &= \delta x_0 + \delta\Delta x = \left(H_0^T R_{z_{C0}}^{-1} H_0\right)^{-1} H_0^T R_{z_{C0}}^{-1} \delta z_{C0} \\ &\quad + \left(H^T R_{z_{PD\phi}}^{-1} H\right)^{-1} H^T R_{z_{PD\phi}}^{-1} \delta z_{PD\phi} \\ &= \left(S_{R_{Z_{C0}}} H_0^T R_{z_{C0}}^{-1} + S_{R_{Z_{PD\phi}}} H^T R_{z_{PD\phi}}^{-1} \Delta H\, S_{R_{Z_{C0}}} H_0^T R_{z_{C0}}^{-1}\right)\nu_{C0} \\ &\quad + S_{R_{Z_{PD\phi}}} H^T R_{z_{PD\phi}}^{-1} \Delta\nu_\phi \\ &\quad + \left(S_{R_{Z_{C0}}} H_0^T R_{z_{C0}}^{-1} + S_{R_{Z_{PD\phi}}} H^T R_{z_{PD\phi}}^{-1} \Delta H\, S_{R_{Z_{C0}}} H_0^T R_{z_{C0}}^{-1}\right) b_{VF} \\ &\quad + S_{R_{Z_{PD\phi}}} H^T R_{z_{PD\phi}}^{-1} f \\ &= B\nu_{C0} + C\Delta\nu_\phi + Bb_V + Cf \end{aligned} \tag{48}$$

where $\Delta H = [\Delta h^1 \; \dots \; \Delta h^m]^T$ and matrices A, B and C have been introduced for the sole purpose of simplifying the notation in the following section, and their definition is obvious from the equation (48).

Protection levels

We will now develop the algorithms to obtain four protection levels, satisfying equations (5) and (6) for the FFC and FDC hypotheses and each of the two implementations presented: $PL_{FFC(RD)}$, $PL_{FFC(PD)}$, $PL_{FDC(RD)}$ and $PL_{FDC(PD)}$.

The starting point to generate the protection levels are the position error formulas (47) and (48). Each of these formulas has terms originated by errors that can be modeled as Gaussian, terms originated by non-Gaussian errors b_V , and for the FDC cases, a term caused by failure f .

The distribution of b_V is unknown, but it is known from (9) that the probability of any $b^i > \beta^i$ is negligible. Assuming further that each b^i is independent of any other error source from the same (or different) satellite, we can conservatively bound the effect of non Gaussian errors in the RD case (47) by

$$Ab_V \le |A|\,|b_V| \le |A|\beta_V \tag{49}$$

where

$$\beta_V = \left[\beta^1 \quad \cdots \quad \beta^n\right]^T \tag{50}$$

and the notation $|\bullet|$ denotes element-wise absolute value operations for the vector b_v and matrix A (not a determinant). Similarly, for the PD case (48):

$$Bb_V \le |B|\,|b_V| \le |B|\beta_V \tag{51}$$

To compute the FFC *PL*s we must then add the effect of Gaussian errors terms (δx_g). For a hypothetical zero mean Gaussian random variable y with standard deviation σ , we first compute an integrity factor k_{int} such that

$$P\left(|y| > k_{\text{int}}\sigma\right) = P_{HMIreq(FFC)} \,. \tag{52}$$

Then we compute the position error standard deviation for the Gaussian errors terms (δx_g). For the RD implementation:

$$E\left[\delta x_{RDg}\delta x_{RDg}^T\right] = S_{R_{Z_{RD}}} \tag{53}$$

We will write the final results in terms of the Vertical Protection Level (VPL); from (47), (49) and (52) and (53):

$$VPL_{FFC(RD)} = k_{\text{int}}\sqrt{S_{R_{Z_{RD}}(3,3)}} + \left(|A|\,\beta_V\right)_{(3,1)} \tag{54}$$

Computing the position error standard deviation for the Gaussian errors terms in the PD implementation is slightly more complicated. For the PD implementation, from (24), (31) and (48):

$$\begin{aligned} E\left[\delta x_{PDg}\delta x_{PDg}^T\right] &= E\left[\left(\delta x_{0g} + \delta\Delta x_g\right)\left(\delta x_{0g} + \delta\Delta x_g\right)^T\right] \\ &= E\Big[\left(S_{R_{Z_{C0}}} H_0^T R_{z_{C0}}^{-1} \delta z_{C0g} + S_{R_{Z_{PD\phi}}} H^T R_{z_{PD\phi}}^{-1} \delta z_{PD\phi g}\right) \\ &\qquad \left(S_{R_{Z_{C0}}} H_0^T R_{z_{C0}}^{-1} \delta z_{C0g} + S_{R_{Z_{PD\phi}}} H_t^T R_{z_{PD\phi}}^{-1} \delta z_{PD\phi g}\right)^T\Big] \\ &= S_{R_{Z_{C0}}} + S_{R_{Z_{C0}}} H_0^T R_{z_{C0}}^{-1} E\left[\delta z_{C0g}\delta z_{PD\phi g}^T\right] R_{z_{PD\phi}}^{-1} H S_{R_{Z_{PD\phi}}} \\ &\quad + S_{R_{Z_{PD\phi}}} H^T R_{z_{PD\phi}}^{-1} E\left[\delta z_{PD\phi g}\delta z_{C0g}^T\right] R_{z_{C0}}^{-1} H_0 S_{R_{Z_{C0}}} + S_{R_{Z_{PD\phi}}} \end{aligned} \tag{55}$$

We assume that the *change* in carrier phase measurement errors over the CT is uncorrelated from the initial code phase errors at time 0. Therefore, using equations (28) through (30), we know that

$$\begin{aligned} E\left[\delta z_{PD\phi g}\delta z_{C0g}{}^{T}\right]_{(i,i)} &= E\left[(\Delta H\delta x_0)\delta z_{C0g}^T\right] \\ &= E\left[\Delta H S_{R_{Z_{C0}}} H_0^T R_{Z_{C0}}^{-1}\delta z_{C0g}\delta z_{C0g}^T\right] \\ &= \Delta H S_{R_{Z_{C0}}} H_0^T R_{Z_{C0}}^{-1} E\left[\delta z_{C0g}\delta z_{C0g}^T\right] \\ &= \Delta H S_{R_{Z_{C0}}} H_0^T \end{aligned} \quad (56)$$

Substituting this result into (55), we obtain the covariance matrix for the Gaussian position error for the PD implementation:

$$E\left[\delta x_{PDg}\delta x_{PDg}^T\right] = S_{R_{Z_{C0}}} + S_{R_{Z_{C0}}}\Delta H^T R_{z_{PD\phi}}^{-1} H\, S_{R_{Z_{PD\phi}}} + S_{R_{Z_{PD\phi}}} H^T R_{z_{PD\phi}}^{-1}\Delta H\, S_{R_{Z_{C0}}} + S_{R_{Z_{PD\phi}}} \quad (57)$$

Using the results from (51) and (57) the resulting FFC VPL equation is:

$$VPL_{FFC(PD)} = k_{\text{int}}\sqrt{E\left[\delta x_{PDg}\delta x_{PDg}^T\right]_{(3,3)}} + \left(|B|\beta_V\right)_{(3,1)} \quad (58)$$

To generate the FDC protection levels, we need to identify the worst failure size and SV combination. Doing this precisely is a time consuming iterative process. A conservative but more practical approach is used here instead. For each SV *i*, a fault magnitude f^{+i} is found such that

$$P\left(r < T \middle| f^{+i}\right) P(FDC) = P_{HMIreq(FDC)} \quad (59)$$

Where

$$r^2 = \left(z_{PD\phi}^+ - H\Delta\hat{x}^+\right)^T R_{z_{PD\phi}}^{-1}\left(z_{PD\phi}^+ - H\Delta\hat{x}^+\right), \quad (60)$$

$$z_{PD\phi}^+ = z_{PD\phi} + \begin{bmatrix} 0 & \cdots & f^{+i} & \cdots & 0 \end{bmatrix}^T, \quad (61)$$

and $\Delta\hat{x}^+$ is the estimate $\Delta\hat{x}$ obtained using $z_{PD\phi}^+$.

We then define a vector, for each satellite *i*, that has as its only non-zero element the value of f^{+i} for that satellite:

$$f_V^i = \begin{bmatrix} 0 & \cdots & f^{+i} & \cdots & 0 \end{bmatrix}^T \quad (62)$$

The resulting VPL must account the impact in the position domain of the various fault vectors f_V^i and also the nominal measurement errors. Furthermore, a fault on the worst case SV, (ie., the satellite fault causing the worst position domain impact) is used to define the FDC VPLs. The results for the RD and PD implementations are, respectively,

$$VPL_{FDC(RD)} = \max_i\left(A f_V^i\right)_{(3,3)} + k_{\text{int}\,FDC}\sqrt{S_{R_{Z_{RD}}(3,3)}} + \left(|A|\beta_V\right)_{(3,1)} \quad (63)$$

$$VPL_{FDC(PD)} = \max_i\left(C f_V^i\right)_{(3,3)} + k_{\text{int}\,FDC}\sqrt{E\left[\delta x_{PDg}\delta x_{PDg}^T\right]_{(3,3)}} + \left(|B|\beta_V\right)_{(3,1)} \quad (64)$$

where $k_{\text{int}\,FDC}$ is a multiplier such that for a zero mean Gaussian distributed random variable *y* with standard deviation σ:

$$P\left(|y| > k_{\text{int}\,FDC}\sigma\right) = \frac{P_{HMIreq(FDC)}}{P(FDC)} \quad (65)$$

The RRAIM detection threshold *T* is set such that the fault-free alarm probability meets the allocated system continuity requirement (P_{Creq}) and is obtained for a fault free non central χ^2 distribution:

$$P\left(r > T \middle| \lambda_{FF}\right) = P_{Creq} \quad (66)$$

The non centrality parameter, λ_{FF} , is obtained using (34) and (48) by treating the non-Gaussian term b_V as a bias:

$$\lambda_{FF} = b_V^T (I - HB)^T R_{z_{PD\phi}}^{-1}(I - HB) b_V \quad (67)$$

But since b_V is unknown, we can obtain an upper bound on λ_{FF},

$$\lambda_{FF} \le \mu \,\|\, \beta_V \,\|^2 \quad (67)$$

where μ is the maximum eigenvalue of $(I - HB)^T R_{z_{PD\phi}}^{-1}(I - HB)$ and $\|\,\beta_V\,\|$ is the magnitude of the bounding vector β_V.

Note that for short CT, the geometry change effect ΔH is generally small, and therefore μ will also be small. In these cases, depending on the magnitude of β_V , it may be acceptable to use a central χ^2 distribution to compute *T*.

Conclusion

The use of Relative Receiver Autonomous Integrity Monitoring (RRAIM) for aircraft precision approach navigation was investigated in this paper. In the concept investigated, the responsibility for detecting hazardously misleading information is divided between the RRAIM-equipped user and the navigation system provider, whose space and ground systems provide the basis for a GNSS Integrity Channel (GIC). The GIC performs ranging source integrity screening, generates corrections, and then broadcasts this information to users worldwide via a space based communication channel. During the correction processing and communication interval, there will be a latent period during which the user must rely on past GIC information. This latency can be a few seconds to several minutes, depending on the implementation. The user is continuously positioning in real time, and integrity against threats occurring during the latency period can be provided by RRAIM.

In this paper two versions of RRAIM were studied: a Range Domain (RD) and a Position Domain (PD) implementation. In both cases the user stores past carrier smoothed code and carrier phase measurements, and selects from those a reference epoch for which it has already received the GIC corrections. In both cases the user has three sets of measurements to use: the stored code measurements with corrections from the reference epoch (whose integrity is ensured by the GIC), a stored carrier phase measurement from the same reference epoch, and the current carrier phase measurement.

In the RD implementation the user creates a set of projected measurements by adding time-differential carrier phase measurements (current minus reference) to the reference GIC-corrected code measurements. The user position is obtained directly using these projected measurements. This implementation and corresponding covariance analysis is relatively straightforward. However, it requires use of only those ranging sources that are continuously available between the current and reference epochs.

The PD implementation generates an initial user position at the reference epoch using the corrected code measurements, and then adds to it a differential position vector, generated with the differential carrier phase measurements. The PD implementation allows for the use of all ranging sources available at the reference time to generate the initial position. The disadvantage is that the covariance analysis is significantly more complicated, and the corresponding bounding protection levels are more difficult to define. This is true because the carrier differential position error includes the effects of changes in user-SV lines of sight. These geometry changes introduce a correlation between the differential (current minus reference) carrier phase position error and the initial code-based reference position error. When latencies of several minutes are considered, these effects cannot be neglected, and have therefore been carefully analyzed and modeled in this paper.

The RRAIM detection function described in this paper is the same for both the RD and the PD implementations, as it needs only detect hazardous measurement errors during the latency period (because for the initial position integrity is provided by the GIC). When general error models for the corrected code measurement errors are considered, including potential unknown biases and Gaussian errors, the geometry change effects introduce a non centrality parameter in the fault free χ^2 distribution of the RRAIM residual. This effect is carefully taken into account in this work.

In summary, this work provides the general formulas are derived for positioning, fault detection, and protection level generation to meet a given set of integrity and continuity requirements. The mathematical justification of assumptions and models is provided, along with practical algorithms that pave the way toward real time implementation.

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