

# Experimental Validation of IMM Algorithm for Carrier Phase Tracking Through Interference

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**Abstract**—In this paper, we develop a carrier phase estimation algorithm to allow GPS receivers to maintain continuous signal tracking through wideband radio frequency interference events. Due to the nature of the GPS signal, carrier phase estimation is a hybrid estimation problem, requiring simultaneous estimation of the discrete-time navigation data bits and the continuous carrier phase. We use an interacting multiple model algorithm with two measurement models corresponding to the two possible choices of navigation data bits (+1 and -1) over each 20 ms coherent integration interval. At the end of each interval, we extract the final state vector estimate by combining the two modes' estimation results using their likelihood functions. We present both simulation and experimental results to evaluate the performance of the phase estimator during wideband interference events.

**Index Terms**—GPS receiver, Wideband Interference, Carrier Phase Tracking, Interacting Multiple Model

## I. INTRODUCTION

In this work, we develop a carrier phase estimation algorithm for GPS receivers that are subjected to wideband radio frequency interference (RFI). This technique allows receiver to effectively track continuous carrier phase and estimate discrete navigation data bits simultaneously. The concept is directly applicable to stationary GPS reference receivers and, when inertial aiding is available, moving receivers as well.

The example application we consider to evaluate performance is a Ground-Based Augmentation System (GBAS) reference station receiver subjected to broadband interference — for example, from nearby use of personal privacy devices (PPDs). Prior work has shown that PPDs most commonly emit broadband interference, and GBAS ground based reference receivers have experienced tracking discontinuities as a result [1]. These events can cause navigation service interruptions to aircraft on final approach. To ensure continuity of the navigation service GBAS reference stations must be able to track GPS signals in the presence of wideband interference.

During an RFI event, as long as existing satellites can be continuously tracked, it is not necessary to acquire new satellites or decode GPS navigation data bits. The reason is that GPS interface specifications ensure that there will always be at least a two-hour overlap of the intervals of applicability between two successive ephemerides broadcast by any satellite. This means that any ephemeris decoded just

prior to the onset of an interference event will be valid for at least two hours after. While the data bits themselves are not of interest, bit transitions must be accounted for in phase estimation.

Tracking GPS signals through wideband RFI requires coherent integration times ( $T_{coh}$ ) longer than the duration of a GPS navigation data bit (20 ms). However, using larger values of  $T_{coh}$  means integrating  $I$  (in-phase) and  $Q$  (quadrature) samples across data bit transitions, effectively averaging them out over time. Potential PLL-based approaches to extend averaging time, such as use of noncoherent memory discriminators [2] or real-time bit estimation [3], introduce biases in the discriminator output, which causes errors in reconstructed carrier Doppler, ultimately leading to cycle slips. A more general problem using PLLs for carrier tracking is that they do not make optimal use of the available information in the GPS signal. This is an especially undesirable characteristic when the signal is already degraded by RFI.

Kalman filter implementations are far more flexible than PLLs [4]. They have been used to track phase through ionospheric scintillation in [5] [6] with  $T_{coh} < 20$  ms and to track weak GPS signals with  $T_{coh} > 20$  ms in [7] by augmenting with Bayesian bit estimation. Kalman filter carrier phase estimation with  $T_{coh} > 20$  ms is a hybrid estimation problem, requiring simultaneous estimation of the discrete navigation data bits and the continuous carrier phase.

In this paper, we develop a new, computationally efficient phase estimator based the Interacting Multiple Model (IMM) algorithm to allow robust carrier phase estimation tracking through sustained RFI. In Section II, the IMM algorithm is explained and the component Kalman filters are set up. Section III derives a dynamic model for clock phase noise dynamics. Section IV shows simulation results of IMM carrier phase estimator. Section V describes the experimental setup and shows the test results.

## II. KALMAN FILTER AND IMM ALGORITHM

While typical PLLs only have fixed structure with predefined loop filters, Kalman filters have internal adaptability to rely either on measurement or phase dynamics more depending on the noise levels.

Extended coherent averaging times are needed to track through interference. If there is no additional operation to deal with navigation data bits, the maximum coherent integration time is 20 ms, which is the duration of one navigation data bit. However, longer coherent integration times can help average out noise tremendously, resulting in much cleaner  $I$  and  $Q$  measurements.

In SDR architectures, carrier phase and code phase tracking loops start with the carrier frequency obtained from the acquisition process. Mixing the raw received signal with local sine and cosine waves at the acquired carrier frequency will produce  $I$  and  $Q$  measurements. Moving forward, the carrier tracking loop will update the local carrier frequency using  $I$  and  $Q$  measurements averaged over the each subsequent coherent integration window. Figure 1 shows the carrier phase tracking architecture in SDR.

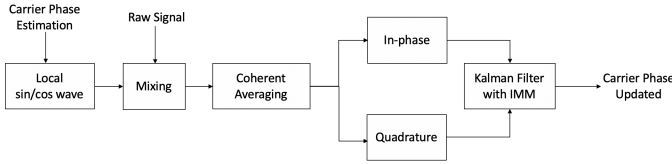


Fig. 1. Carrier phase tracking in SDR

Both the carrier phase and code phase tracking loops rely on the determination of carrier frequency which means the quality of  $I$  and  $Q$  measurements is extremely important. The interference event contributes additive white Gaussian noise (AWGN) directly into  $I$  and  $Q$  measurements. For typical PLL operation, using a simple phase discriminator, these noisy measurements can easily cause phase errors exceeding the pull-in limit of phase discriminator, usually leading to cycle slips and eventual loss of lock.

The shortcomings of a traditional PLL can be overcome using a Kalman filter to estimate phase directly. The goal of any carrier phase tracking loop is to produce best phase estimate under noisy conditions. The Kalman filter does precisely this given that the noise is white, which is the case in wideband interference. In our interference scenario, the receiver is running normally when hit by an RFI event, so Kalman filter estimate error is small at the start.

#### A. Dynamic model

The clock phase noise can be modeled using a  $2^{nd}$  order system. The details of the model will be discussed in Section III.

$$\begin{bmatrix} \dot{\phi}_{clk} \\ \phi_{clk} \end{bmatrix}_{k+1} = \Phi \begin{bmatrix} \dot{\phi}_{clk} \\ \phi_{clk} \end{bmatrix}_k + w_k \quad (1)$$

Over one coherent averaging interval, the total phase change is  $\Delta\phi_{total} = \Delta\phi_{true} + \phi_{clk}$ , where  $\Delta\phi_{true}$  is the phase change due to the relative movement of the satellite relative to the receiver. For our current development, we assume a static receiver – e.g., a GBAS reference receiver – so that

$\Delta\phi_{true}$  is the result of satellite motion only, which is known using the broadcast ephemeris. Later on when we have moving user, the relative motion between satellite and receiver can be accommodated using inertial sensors.

The carrier frequency can be updated using Equation 2:

$$f_{new} = f_{old} + \frac{d}{dt}(\Delta\phi_{true}) + \dot{\phi}_{clk} \quad (2)$$

#### B. Measurement model

The measurement model can be written as

$$I = dA \cos(\Delta\phi_{total}) + v_{i,k} \quad (3)$$

$$Q = dA \sin(\Delta\phi_{total}) + v_{q,k} \quad (4)$$

where  $d$  is the navigation data bit,  $A$  is the signal amplitude,  $v_{i,k}$  and  $v_{q,k}$  are i.i.d.  $\sim N(0, V)$ , where  $V = I\sigma_v^2$  and related to the carrier-to-noise ratio as follows:

$$C/N_0 = 10 \frac{C/N_0(\text{dBHz})}{10} \quad (5)$$

$$\sigma_v = \frac{1}{\sqrt{0.04 C/N_0}} \quad (6)$$

#### C. IMM algorithm

The IMM stands for Interactive Multiple Model, it is a multiple hypothesis estimation algorithm. It assumes a system obeys one of a finite number of models at a time [8]. It is generally composed of the five steps shown in Figure 2. Starting with prior probabilities of each model being correct, parallel Kalman updates are executed for each, and the mode likelihood functions are evaluated based on the measurements. Then post measurement mode probabilities are determined, and the results from all of the modes are weighted and combined to produce the output state vector estimate and its error covariance matrix.

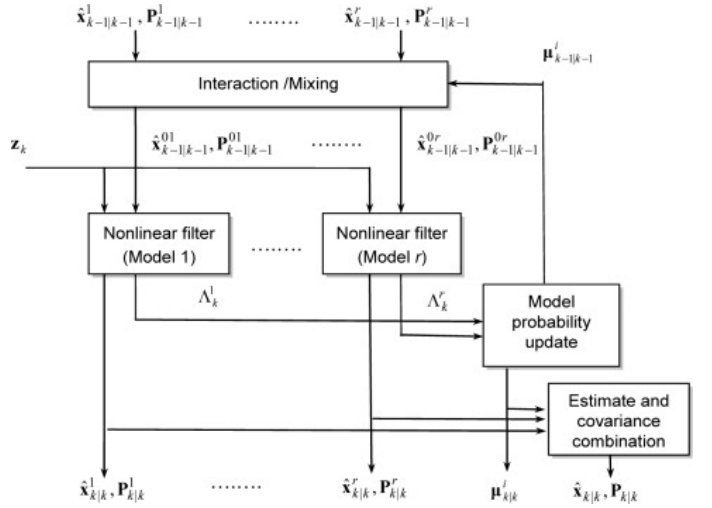


Fig. 2. IMM algorithm

In our IMM application, as shown in Figure 3, two modes run in parallel, corresponding to the two navigation data bits

values, 1 and  $-1$ . The mode (bit) transition probabilities are  $\frac{1}{2}$  and  $\frac{1}{2}$ , which means the data bits are sequentially independent of each other. However, the data bits in reality are not totally random. The knowledge of some navigation data bits can be utilized as we shown in our prior work [9]

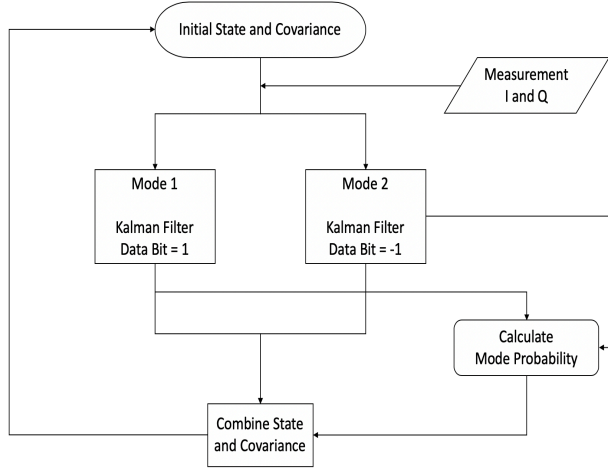


Fig. 3. IMM algorithm in our case

### III. CLOCK DYNAMIC MODEL

We use Rubidium atomic oscillators (SpecrtaTime LPRFS) for the receiver and GPS signal simulator used in this work. The random fluctuations in phase, corresponding to jitter in time domain, is described by the clock's phase noise power spectral density (PSD), which is modeled as:

$$S(f) = h_2 f^0 + h_1 f^{-1} + h_0 f^{-2} + h_{-1} f^{-3} + h_{-2} f^{-4} \quad (7)$$

However, the clock specifications did not cover the phase noise PSD in lower frequency ranges needed for our purpose. So we need to derive a set of  $h$  coefficients from Allan variance data.

#### A. Full Frequency Range Plot

The Allan variance plot measures the short term stability of the clock, which is shown as the blue line in Figure 4.

The slopes of the blue line can be approximately matched to the terms in Equation 7 using three asymptotes from  $h_1, h_0$  and  $h_{-1}$ . Using the relations shown in Figure 5 from [10], the following coefficients are selected:  $h_2 = 0$ ,  $h_1 = 1.000 \times 10^{-22}$ ,  $h_0 = 3.401 \times 10^{-23}$ ,  $h_{-1} = 4.617 \times 10^{-27}$ ,  $h_{-2} = 0$ .

#### B. Transfer Function Bound and Discretization

The clock phase noise PSD is evaluated using Equation 7 with the three nonzero  $h$  coefficients and shown as the blue stars in Figure 6. For simplicity in the estimator implementation, we model the clock phase noise as white noise fed through a second order filter with transfer function  $\frac{25}{s^2 + 100s}$ , which is shown as the black line in Figure 6.

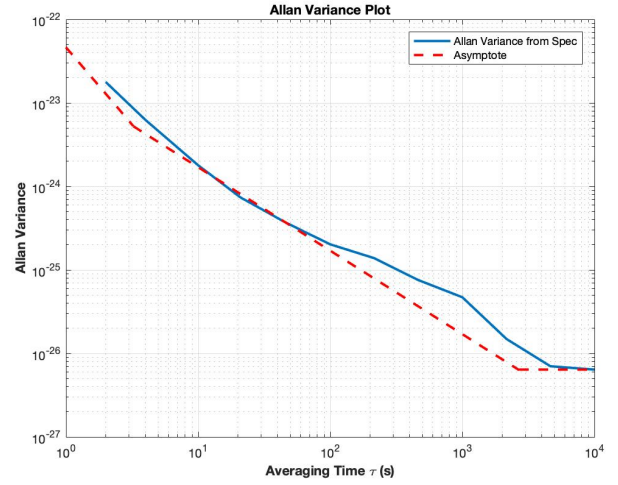


Fig. 4. Allan variance asymptotes

Description of noise process	$S_y(f) =$	$S_\delta(f) =$	$\sigma_y^2(\tau) =$
Random walk FM	$h_{-2} f^{-2}$	$h_{-2} \nu^2 f^{-4}$	$A h_{-2} \tau^1$
Flicker FM	$h_{-1} f^{-1}$	$h_{-1} \nu^2 f^{-3}$	$B h_{-1} \tau^0$
White FM	$h_0 f^0$	$h_0 \nu^2 f^{-2}$	$C h_0 \tau^{-1}$
Flicker PM	$h_1 f^1$	$h_1 \nu^2 f^{-1}$	$D h_1 \tau^{-2}$
White PM	$h_2 f^2$	$h_2 \nu^2 f^0$	$E h_2 \tau^{-2}$

$$A = \frac{2\pi^2}{3} \quad B = 2 \ln 2 \quad C = 1/2$$

$$D = \frac{1.038 + 3 \ln(2\pi f_b \tau)}{4\pi^2} \quad E = \frac{3f_b}{4\pi^2}$$

Fig. 5. Allan variance calculation [10]

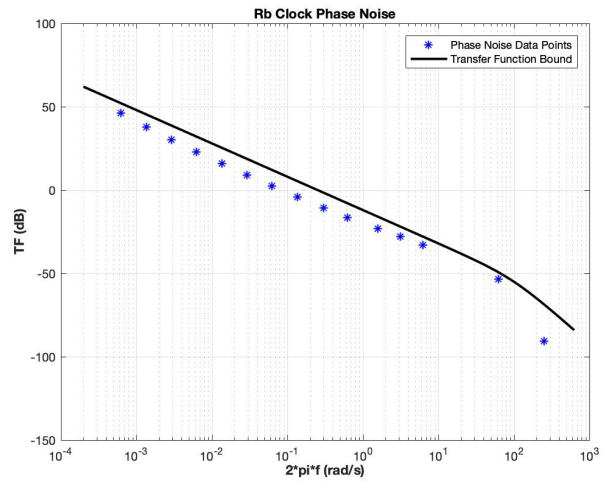


Fig. 6. Transfer function bound

To implement clock dynamics in the IMM's Kalman filters, we write the transfer function in state space form. While there are infinite number of choices when converting a transfer

function to state space realization, we need to ensure that the output for the state space representation should be the phase itself. We choose the simple realization below, where phase and frequency form the two-dimensional state vector.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -100 \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} 0 \\ 25 \end{bmatrix} u \quad (8)$$

$$\phi = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix} \quad (9)$$

Then we use Van-Loan's method to discretize Equation 10,

$$\begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix}_{k+1} = \begin{bmatrix} 1 & 0.0086 \\ 0 & 0.1353 \end{bmatrix} \begin{bmatrix} \phi \\ \dot{\phi} \end{bmatrix}_k + w_k \quad (10)$$

where the covariance matrix for  $w_k$  is

$$W = \begin{bmatrix} 0.0005 & 0.0234 \\ 0.0234 & 3.0678 \end{bmatrix} \quad (11)$$

#### IV. SIMULATION RESULTS

Our simulations focus on evaluating IMM Kalman filter tracking performance in low  $C/N_0$  conditions while accounting for clock dynamics and unknown navigation data bits. When hit by interference, the noise level on the  $I$  and  $Q$  samples greatly increases. Compared to the thermal noise, clock phase noise is relatively small, but it can be a long term, accumulative error source.

In our prior work [9], we simulated the operation of the IMM algorithm with a constant fixed phase to verify the hybrid estimation solutions. We also tested performance when a certain fraction of navigation data bits are assumed known, because the navigation data bits are actually not a totally random sequence of +1 and -1. (For example, the preambles are always the same fixed and the sequence representing the GPS week is quite likely to stay the same during the interference event. [11])

Here we simulate an interference event that drops the carrier-to-noise ratio from 45 dB-Hz to 15 dB-Hz. The coherent averaging time for each  $I$  and  $Q$  measurement pair is 20 ms and the total simulation time was 6 sec (300 Kalman steps). The effects of clock phase noise are included. One hundred independent runs were simulated.

We assume that that approximately  $\frac{1}{3}$  of navigation data bits are predictable in groups. We model this by making the first  $\frac{1}{9}$ , the fourth  $\frac{1}{9}$  and the seventh  $\frac{1}{9}$  of bits known. So the total fraction of known bits is still  $\frac{1}{3}$ . The results are shown in Figure 7.

The figure shows phase estimate error in degrees versus time. The colored lines represent the 100 individual runs, and the black thick lines are the corresponding sample standard deviations of phase tracking error over time. The standard deviation of the phase tracking error reaches steady state quickly and stays at about 9.6 degrees for entire tracking period. The actual standard deviation of colored lines is 9.8 degrees which match with the black lines. When the Kalman

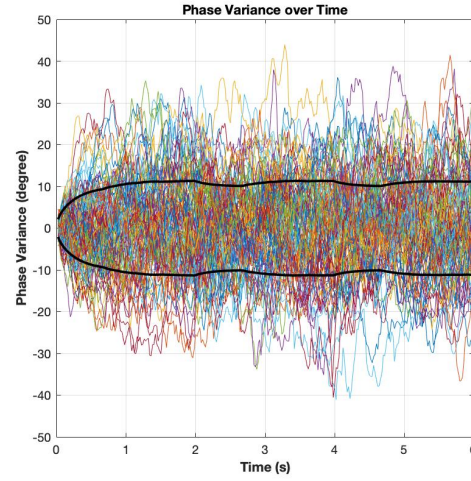


Fig. 7. Simulation result

filter knows navigation data bits, the colored lines begin to narrow down and standard deviations shrink. In comparison, a standard PLL loses lock immediately at the start of the RFI event ( $t = 0$ ).

#### V. EXPERIMENTAL DATA VALIDATION

##### A. Experimental Setup

Experimental data is generated using a Spectracom GSG-6 GNSS simulator. A USRP N200 is used as the front end and the IMM algorithm implemented in a software defined receiver (SDR).

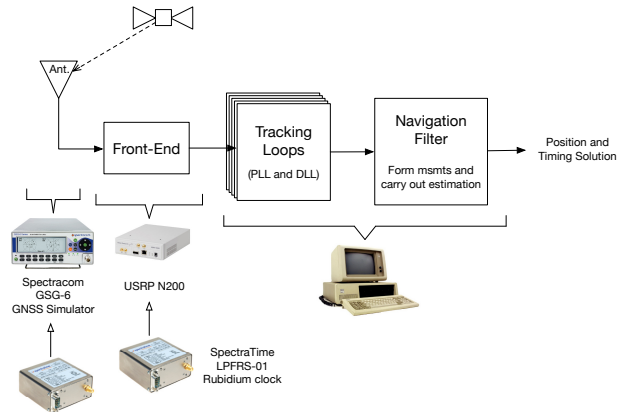


Fig. 8. Experimental setup

The experimental scenario covers a static GPS reference receiver over a 4 min period. High elevation satellites are used in this experimental test. The tracking ability of our IMM filter should be the same as typical PLL under normal condition scenario (45 dB/Hz).

In the experimental test, we running typical PLL for 20 seconds then switch to IMM filter for the rest of time.

## VI. RESULT AND ANALYSIS

Figure 9 shows the tracking result for PRN 30. Tracking error in degrees is plotted versus time in milliseconds.

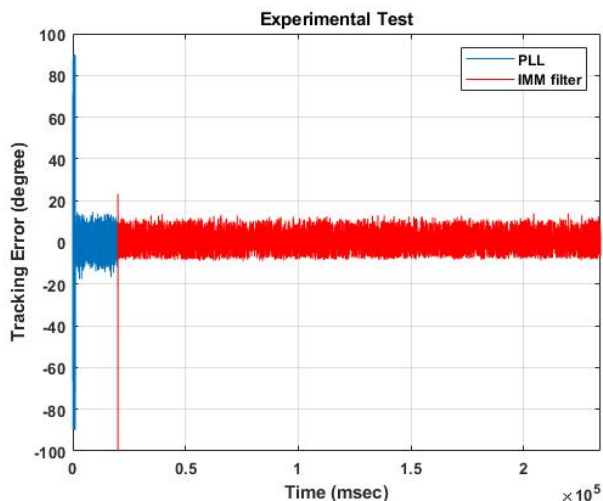


Fig. 9. Tracking result comparison

At the moment of 20 seconds, we switch to IMM filter tracking. The IMM filter achieves reliable tracking at 45 dB/Hz, even slightly better than the typical PLL in terms of variance of tracking error.

In the next phase of our work we will experimentally evaluate performance during an interference event by dropping power out of the signal simulator.

## VII. CONCLUSION

In this paper, we develop an IMM carrier phase estimation algorithm to allow GPS receivers to maintain continuous signal tracking through wideband radio frequency interference events.

The IMM algorithm and its component Kalman filters were shown to work well for carrier phase tracking in both simulations and experiments. The results show a 9.6 degrees phase standard deviation for 15 dB-Hz carrier-to noise ratio in simulation. The experimental results so far show that the IMM is able to track as well as (or slightly better than) a traditional PLL under normal signal levels (no interference).

In future work, navigation data bit structure information will be used. Adaptive noise variance estimation will also be added to the IMM. The transition matrix between each noise level needs to be modeled in this case. The algorithm will also be applied to moving receivers with Doppler aiding from inertial sensors.

## ACKNOWLEDGMENT

We would like to thank our sponsors at the Federal Aviation Administration (FAA) for supporting this research. The views and opinions expressed in this paper are those of the authors and do not necessarily reflect those of any other organization or person.

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