LAAS Ranging Error Overbound for Non-zero Mean and Non-gaussian Multipath Error Distributions

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BIOGRAPHY

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ABSTRACT

Providing sufficient navigation integrity is one of the most critical elements in the development Local Area Augmentation System (LAAS) architecture and remains an unsolved key technical problem. For example, in LAAS, the navigation integrity is ensured through the computation of the aircraft position error bounds (i.e., Protection Levels, VPL/LPL) in which implicitly zero-mean, normally distributed fault-free error models are assumed. However, it is understood that true ranging error cannot easily be described in this way, especially in the presence of ground reflection multipath. Therefore, to ensure navigation integrity of LAAS, the true ranging error must be *bounded* with zero-mean normal distribution. In this regard, a major ranging error source, ground reflection multipath, is specifically targeted in this research and new theoretical approaches are defined to bound and account for its effects.

INTRODUCTION

The Local Area Augmentation System (LAAS) is a differential satellite-based navigation system architecture designed to provide navigation services for civil aircraft users during precision approach and landing. In the design of architecture, one of the most challenging and remaining unsolved technical problems is to provide sufficient integrity-related broadcast information (i.e., pseudorange broadcast correction error standard deviation, σ_{pr_gnd}) to aircraft. The integrity-related information is translated to integrity risk assessment through the computation of Vertical and Lateral Protection Levels (termed VPL and LPL, respectively) to determine whether or not LAAS is available to support precision approach and landing. These levels are the position bounds that can be ensured with an acceptable level of integrity risk. The prescribed algorithms for the generation of the Protection Levels assume a zero-mean normally distributed fault-free error model for the broadcast pseudorange corrections. The standard deviation of correction error is presumed by the aircraft to be equal to the broadcast value of σ_{pr_gnd} for each satellite. In reality, while such an error distribution model appears to be reasonably consistent with reference receiver-related ranging errors due to receiver noise and diffuse multipath, remaining errors such as ground reflection multipath and systematic reference receiver/antenna errors may be non-gaussian, non-zero-mean, or both. Therefore, LAAS must take special care when generating σ_{pr_gnd} to guarantee integrity risk at desired level of probability.

For normally distributed error sources such as receiver-related noise and diffuse multipath, standard deviations can be estimated using experimental data alone. In this case, however, it is still necessary to account for the additional integrity risk incurred by statistical uncertainty (due to finite sample size) in the knowledge of reference receiver error standard deviation and error correlation between multiple reference receivers. In prior work [3], a detailed methodology was proposed for the definition of minimum acceptable inflation parameters for the sample standard deviation. Furthermore, in order for such an empirical process to be applied, it is necessary to define a proper method to collect data into bins prior to sigma estimation. While large bin sizes are desired to maximize sample size (to limit required inflation factors), bin size is ultimately constrained by the need for spatial stationary of all data within the bin (i.e., all error data within a bin must have the same underlying distribution). One candidate approach toward the resolution of this critical tradeoff is an adaptive estimation technique known as Expanding Bin method (EB-method) [9].

Bias-type error sources, such as systematic reference receiver/antenna errors must be accounted for by an appropriate method of bounding a non-zero mean distribution with a zero mean normal distribution. Since LAAS Ground Facility (LGF) mean error values will not be broadcast to the aircraft by the LGF, the effect of a non-zero mean must be accounted for by further inflation of σ_{pr_gnd}. However, to ensure tight position domain (VPL and LPL) overbounds at the aircraft in the presence of a non-zero mean, the value of the required inflation factor on σ_{pr_gnd} is dependent on airborne error statistics (σ_{pr_air}). Clearly prior information of aircraft error statistics will be unavailable at the LGF, so in this paper we introduce new methods for σ_{pr_gnd} inflation (due to non-zero mean), which result in looser (i.e., more conservative) position domain bounds, but do not require any specific prior knowledge of aircraft error statistics.

For non-gaussian error sources, such as ground reflection multipath, standard deviations cannot be estimated by the direct use of data because empirically computed (and inflated) values of σ_{pr_gnd} are not sufficient to guarantee overbounding of the total LGF ranging error. Furthermore, it is impossible to rely on empirically constructed distributions (e.g., error data histograms) alone to define the nature of the underlying error distribution because little or no empirical data will exist in the 'tails' (which are of greatest interest in LAAS). Therefore, theoretical approaches are sought in our work to incorporate non-gaussian error effects into σ_{pr_gnd}. In this regard, ground reflection multipath error, which is the largest contributor source to ranging error, is of particular interest. Theoretical solutions are investigated for modeling ground reflection multipath error sigmas. It is also realized that for non-gaussian error distributions, position domain error bounding must be specifically addressed because a gaussian bounding that holds in the range domain (single-satellite error source) does not necessarily guarantee similar bounding in the position domain (multiple-satellite error sources in weighted least square solution). Therefore, position domain bounding must be verified via multiple convolutions of non-gaussian distributions.

A practical way is then introduced to synthesize the empirical and theoretical elements described above to quantitatively establish σ_{pr_gnd} for LAAS. In particular, the approach taken for sigma establishment is to compute a best estimate of sigma from data using EB-method, and then combine the result with those obtained by theoretical means for ground reflection multipath. It is clear that, however, that the theoretical models must be consistent with the underlying physical processes driving the errors. In order to avoid detailed validation of physical ground reflection multipath models, we consider in this work two limit cases model distributions for the overbounding of ground reflection multipath. These are a worst-case bias-type distribution and a worst-case symmetric, zero-mean, bi-modal distribution. The effectiveness of synthesized sigma results is demonstrated using analysis, simulation and experimental data.

BOUNDING CONCEPT

An overbound for aircraft position error is needed when the true ranging error distributions are not zero-mean gaussian because, in LAAS, the aircraft assumes that the broadcast sigma is a standard deviation of a zero-mean gaussian probability density function (PDF). It is the responsibility of the LGF to provide a standard deviation of a zero-mean gaussian PDF that represents the true correction error distribution. In reality, however, situations may exist in which *non-gaussian* and/or *non zero-mean gaussian* distributions must be bounded.

Bounding for ranging error is defined as a method by which a sufficient zero-mean gaussian distribution is generated to overbound the tails of true aircraft position errors having non-gaussian or non-zero mean gaussian PDFs. To ensure the validity of bounding in the position domain, method involves the convolutions of multiple non-gaussian PDFs (i.e., sum of non-gaussian RVs) and the convolution of the same number of gaussian PDFs. The resulting tail area of Cumulative Distribution Functions (CDFs) is evaluated to determine whether the gaussian result overbounds the non-gaussian at the probability level of interest. The ratio of overbounding gaussian standard deviation to that of the non-gaussian PDF parameter is called the *Inflation Factor*.

MEAN BOUNDING FOR LAAS

As mentioned above, bias values in ranging error are neither broadcast to the aircraft nor defined as a part of Protection Level equations. Therefore, the existence of such biases must be accounted for by sigma overbounding. Overbounding non-zero mean gaussian distributions is easy in principle, but difficulties arise when the LGF and aircraft are unaware of distribution parameters corresponding to each other's error PDFs. This difficulty limits LGF ability to know whether or not a meaningful protection level is computed at aircraft prior its broadcast values. For example, broadcast sigmas of correction error are generated at the LGF, broadcast to the aircraft, combined with sigmas of the aircraft, and then used in Protection Level equations at aircraft for the final navigation integrity risk assessment of precision landing. Both LGF and aircraft error statistics collectively must be considered in the mean bounding process. To *precisely* accommodate for the existence of a mean value in the correction error, the LGF must know aircraft ranging error distribution in order to generate an appropriate overbounding value of σ_{pr_gnd}. Unfortunately, the LGF obviously cannot know anything about the incoming aircraft errors. The critical problem, then, in the bounding process is to account for non-zero LGF mean errors without prior knowledge of each other's error distribution parameters.

It is also noted that the mean values do not necessarily always have positive values. In reality, however, it is often only possible to define a bound on the absolute value of the mean ranging error. Therefore, for a conservative bound, the mean values can be assumed to be in the same direction (absolute values).

As described in [1] and [4], the computation of Protection Levels is based on a Weighted Least Square Estimation, which is driven by the satellite geometry, and ranging error covariance. In VPL, the sigmas are scaled with projection matrix elements. These elements change with satellite geometry. Therefore, bounding is not a pure function of distribution parameters. Satellite geometries also add further complexity in bounding process. Let us recall the VPL equation and make a simple modification by grouping the error sources in terms of their association with LGF and aircraft,

$$VPL = k\sqrt{\sum_{n=1}^{N} S_{zn}^2\left(\sigma_{gnd,n}^2 + \sigma_{air,n}^2\right)} \qquad (1)$$

where, $k \equiv k_{md\_ff}$, $\sigma_{gnd,n}^2 \equiv \sigma_{pr\_gnd,n}^2 / M$, and $\sigma_{air,n}^2 \equiv \sigma_{pr\_air,n}^2 + \sigma_{pr\_res,n}^2$. Now, the *true* protection level Equation (1) including biases can be expressed as VPL_{act} in the following form:

$$VPL_{act} = k\sqrt{\sum_{n=1}^{N} S_{zn}^2\left(\bar{\sigma}_{gnd,n}^2 + \bar{\sigma}_{air,n}^2\right)} + \sum_{n=1}^{N} S_{zn}\left(\left|\bar{\mu}_{gnd,n}\right| + \left|\bar{\mu}_{air,n}\right|\right) \qquad (2)$$

where, $\bar{\sigma}$ and $\bar{\mu}$ denote true values of sigmas and means describing the true error PDFs for the LGF and aircraft. The aircraft position error bound is desired and integrity is maintained if $VPL \geq VPL_{act}$ holds. Expanded form of this inequality can be factored as follows,

$$\sum_{n=1}^{N} S_{zn}^2\left(\sigma_{gnd,n}^2 + \sigma_{air,n}^2\right) \geq \left(1 + \frac{\sum_{n=1}^{N} S_{zn}\left(\left|\bar{\mu}_{gnd,n}\right| + \left|\bar{\mu}_{air,n}\right|\right)}{k\sqrt{\sum_{n=1}^{N} S_{zn}^2\left(\bar{\sigma}_{gnd,n}^2 + \bar{\sigma}_{air,n}^2\right)}}\right)^2 \sum_{n=1}^{N} S_{zn}^2\left(\bar{\sigma}_{gnd,n}^2 + \bar{\sigma}_{air,n}^2\right) \qquad (3)$$

We desire to define $\sigma_{gnd,n}$ and $\sigma_{air,n}$ to ensure aircraft position error is bounded at desired level of probability of interest (defined by value of k). Such a bounding is ensured if we require that the following inequalities hold for each satellite,

$$\sigma_{gnd,n} \geq \xi\bar{\sigma}_{gnd,n}, \qquad \sigma_{air,n} \geq \xi\bar{\sigma}_{air,n} \qquad (4a, 4b)$$

where the *Inflation Factor* (ξ), a common multiplier for both LGF and aircraft sigmas, is

$$\xi = \left(1 + \frac{\sum_{n=1}^{N} S_{zn}\left(\left|\bar{\mu}_{gnd,n}\right| + \left|\bar{\mu}_{air,n}\right|\right)}{k\sqrt{\sum_{n=1}^{N} S_{zn}^2\left(\bar{\sigma}_{gnd,n}^2 + \bar{\sigma}_{air,n}^2\right)}}\right) \qquad (5)$$

It is easily observed that the Inflation Factor (ξ) is a nonlinear function of the true sigma and mean values of both LGF and aircraft. Since ξ includes all statistical parameters, the bounding of mean values becomes a difficult task because the LGF is unaware of aircraft statistics prior sigma broadcast. Therefore solutions are sought that do not depend on knowledge of aircraft error distribution parameters. Two such solution approaches are examined here for accommodation of the possible existence of mean values: 1) a bounding is sought without any specific information regarding LGF and/or aircraft error PDFs, 2) a direct accommodation of mean values is sought with Alert Limit (i.e., VAL) buffering.

A bounding method for ground error is introduced here to account for lack of aircraft information in the bounding process. For example, let us recall bounding inequality, expand and rewrite it in the vector form as follows,

$$\|\sigma\|_2 \geq \left(\|\bar{\sigma}\|_2 + \tfrac{1}{k}\|\bar{\mu}\|_1\right) \qquad (6)$$

where, $\sigma = \left[S_{z1}\sigma_{gnd,1} \quad S_{z1}\sigma_{air,1} \quad S_{z2}\sigma_{gnd,2} \quad \ldots \quad S_{zN}\sigma_{air,N}\right]_{(1\times 2N)}$, $\bar{\sigma} = \left[S_{z1}\bar{\sigma}_{gnd,1} \quad S_{z1}\bar{\sigma}_{air,1} \quad S_{z2}\bar{\sigma}_{gnd,2} \quad \ldots \quad S_{zN}\bar{\sigma}_{air,N}\right]_{(1\times 2N)}$, $\bar{\mu} = \left[S_{Z1}\bar{\mu}_{gnd,1} \quad S_{Z1}\bar{\mu}_{air,1} \quad S_{Z2}\bar{\mu}_{gnd,2} \quad \ldots \quad S_{Zn}\bar{\mu}_{air,N}\right]_{(1\times 2N)}$, and N is the maximum number of satellites in view (seen by the LGF) at the time of sigma broadcast. The bias term can be bounded as (with its second norm) [5] $\sqrt{2N}\|\bar{\mu}\|_2 \geq \|\bar{\mu}\|_1$. Applying this relation to inequality (6) results in the following expression where we impose a more conservative requirement on $\|\sigma\|_2$.

$$\|\sigma\|_2 \geq \left(\|\overline{\sigma}\|_2 + \tfrac{\sqrt{2N}}{k}\|\overline{\mu}\|_2\right) \geq \left(\|\overline{\sigma}\|_2 + \tfrac{1}{k}\|\overline{\mu}\|_1\right) \qquad (7)$$

An even more conservative, but simpler bound can be defined as:

$$\|\sigma\|_2 \geq \sqrt{2}\left(\left(\|\overline{\sigma}\|_2\right)^2 + \left(\tfrac{\sqrt{2N}}{k}\|\overline{\mu}\|_2\right)^2\right)^{\frac{1}{2}} \geq \left(\|\overline{\sigma}\|_2 + \tfrac{\sqrt{2N}}{k}\|\overline{\mu}\|_2\right) \geq \left(\|\overline{\sigma}\|_2 + \tfrac{1}{k}\|\overline{\mu}\|_1\right) \quad (8)$$

For example, let us now expand this inequality,

$$\sqrt{\sum_{n=1}^{N} S_{zn}^2\left(\sigma_{gnd,n}^2 + \sigma_{air,n}^2\right)} \geq \sqrt{2}\sqrt{\sum_{n=1}^{N} S_{zn}^2\left(\overline{\sigma}_{gnd,n}^2 + \overline{\sigma}_{air,n}^2\right) + \tfrac{2N}{k^2}\sum_{n=1}^{N} S_{zn}^2\left(\overline{\mu}_{gnd,n}^2 + \overline{\mu}_{air,n}^2\right)} \quad (9)$$

and then establish detailed geometry independent mean bounding models from (9) as discussed next.

Equation (9) is the most general form of bounding because it assumes that the mean values exist not only at LGF but also at aircraft. To date, the sigma generation and error processing for aircraft ranging error is not finalized and it is yet unclear whether biases will exist in the aircraft errors. In this analysis we consider all possible scenarios, including aircraft biases as described in the following cases.

Case-A: Mean values exist at the both LGF and Aircraft. (i.e., $\overline{\mu}_{gnd,n} \neq 0$ and $\overline{\mu}_{air,n} \neq 0$). In this case, the following expressions can be written for the LGF broadcast and aircraft sigmas respectively,

$$\sigma_{gnd,n} \geq \sqrt{2}\sqrt{\overline{\sigma}_{gnd,n}^2 + \tfrac{2N}{k^2}\overline{\mu}_{gnd,n}^2}\ ,\ \sigma_{air,n} \geq \sqrt{2}\sqrt{\overline{\sigma}_{air,n}^2 + \tfrac{2N}{k^2}\overline{\mu}_{air,n}^2} \quad (10a, 10b)$$

It is clear that inequality (9) is preserved by (10a. 10b) if it is applied to each satellite individually.

Case-B: Mean values exist only at the LGF (i.e., $\overline{\mu}_{gnd,n} \neq 0$ and $\overline{\mu}_{air,n} = 0$). In this case, the LGF and aircraft sigmas can be expressed as,

$$\sigma_{pr\_gnd,n} \geq \sqrt{2}\sqrt{\overline{\sigma}_{gnd,n}^2 + \tfrac{N}{k^2}\overline{\mu}_{gnd,n}^2}\ ,\quad \sigma_{pr\_air,n} \geq \sqrt{2}\overline{\sigma}_{air,n} \quad (11a\ 11b)$$

Case-C: Mean values exist only at the Aircraft (i.e., $\overline{\mu}_{gnd,n} = 0$ and $\overline{\mu}_{air,n} \neq 0$). In addition to the above two sub-cases, the biases may exist only at the aircraft. In this case the inflation of the ground sigmas is unnecessary.

The results, summarized in Table 1 for *Case-A* and *Case-B* are based on range domain bounds for the LGF and aircraft, respectively. As shown, these range domain bounds are geometry-independent and sufficient to ensure a position error overbound. We can thus accommodate range error biases on individual satellites simply by assuming a zero mean and inflating sigma appropriately. The cost of such practicality and simplicity is an additional inflation on sigmas of aircraft even in the absence of aircraft mean values. In other words, even if biases do not exist at the aircraft we still need to inflate aircraft sigmas by $\sqrt{2}$ in order to ensure a position domain error bound. However a lower inflation may be possible in some special cases. For example, in both Case A and Case B, N is assumed as maximum number of satellites in view; however, N need only be associated with the number of satellites whose distributions exhibit a mean value. Such an assumption reduces the unnecessary inflation of LGF sigmas if some satellites in view do not have biases. However, even in this case, the aircraft still needs to inflate its own sigmas by a factor of $\sqrt{2}$.

Summary of Mean Bounding Model. A mean bounding model is proposed. The model consists of two sub-cases, corresponding to scenarios where the biases exist at the LGF and/or at the aircraft. It is shown that this model conservatively bounds biases; therefore it is appropriate from the perspective of integrity. The significance of the Model is the bound does not require any specific prior knowledge of aircraft error statistics. The summary is shown in Table 1.

Table 1 Summary of Mean Bounding

	LGF	**Aircraft**
Case A	$\sigma_{pr\_gnd,n} \geq \sqrt{2}\sqrt{\overline{\sigma}_{gnd,n}^2 + \tfrac{2N}{k^2}\overline{\mu}_{gnd,n}^2}$	$\sigma_{pr\_air,n} \geq \sqrt{2}\sqrt{\overline{\sigma}_{air,n}^2 + \tfrac{2N}{k^2}\overline{\mu}_{air,n}^2}$
Case B	$\sigma_{pr\_gnd,n} \geq \sqrt{2}\sqrt{\overline{\sigma}_{gnd,n}^2 + \tfrac{N}{k^2}\overline{\mu}_{gnd,n}^2}$	$\sigma_{pr\_air,n} \geq \sqrt{2}\overline{\sigma}_{air,n}$

GROUND REFLECTION MULTIPATH

Multipath is a well-known cause of signal tracking error not only for GPS but also for all types of Radio Frequency (RF) applications. It is caused by undesired reflected/diffracted signals from various surfaces in the near vicinity of the antenna. These surfaces can be the ground or other objects such as a building, a moving vehicle, or a tree. In general, due to the slowly varying nature of such environmental factors, it is unlikely that the effect of multipath error distributions can be quantified by experimental means alone. Collecting error data samples over many days (for example, for one year over all four seasons) may not reliably describe all variations of the error. There are two reasons for this: 1) sample sizes are limited and can be highly correlated over adjacent days;

therefore, information about distribution tails cannot be easily extracted from a limited set of samples, and 2) for practical LGF initialization purposes it is impossible to rely on a long term error collection prior to system initialization.

Ground reflection multipath also can be resistant to filtering. For example, satellites at the same elevation but with different angular rates will have different multipath errors because for satellites with higher angular rates, carrier aided smoothing of pseudorange measurements will for the most part mitigate multipath. This phenomenon can be explained by the noise attenuation performance of the smoothing filter, which is a function of filter bandwidth.

The persistency of ground reflection multipath error after the carrier-smoothing filter can be shown easily. For example, multipath error may be characterized by its Doppler rate as $\dot{\theta} = (4\pi h/\lambda)\dot{E}\cos E$. Where, h is the antenna height, λ is the GPS wavelength of 19 cm, and E and $\dot{E}$ are elevation and elevation rate, respectively.

The bandwidth of the carrier smoothing filter loop is $1/2\tau$ Hz or $\pi/2\tau$ rad/s, where τ is the smoothing filter time constant. For LAAS, $\tau = 100$ second, so the bandwidth of the filter loop is 0.0314 rad/s. If the filter bandwidth is higher than the multipath Doppler rate then the smoothing filter is unable to attenuate the multipath error. [7].

An example of noise attenuation versus antenna height is performed by simulation the GPS constellation of 24 satellites at O'Hare International Airport, Chicago. The results of this simulation show that an antenna height of > 5 meters is required for the smoothing filter to reliably attenuate errors caused by the ground reflection multipath. For comparison, the LAAS Test Prototype antennas heights are approximately 2 meters, so most ground reflection multipath will not attenuated by the filter.

It is noted that a discrete object's reflection multipath error can be modeled in a similar manner but the delays are usually longer because of the larger reflection distances between antenna and reflection surfaces (relative to ground reflection). Therefore, the Doppler rate for such multipath naturally is high. Secondly, an important characteristic of discrete object reflection multipath is the duration of reflected signals. The satellite position and reflection surface orientations may produce a specific geometry that contributes multipath for a longer duration of reflected signal but at a faster Doppler rate. In contrast, some geometries may contribute shorter durations of multipath and slower Doppler rates. In either case, the filter smoothing can successfully average out these types of error. The ground reflection multipath, unlike these cases, contains both disadvantages: longer duration of multipath and very slow Doppler rate. In this research, therefore, theoretical approaches are emphasized only for ground reflection multipath error. Two candidate distribution models are proposed for the establishment and inflation of correction error standard deviation to account for the effect of ground reflection multipath.

In this analysis, a Multipath Limiting Antenna (MLA) implementation is assumed at the LGF. The MLA consists of a top ('high zenith') antenna for tracking high elevation satellite and a bottom antenna (Dipole) for tracking low elevation satellites. The primary driver in the MLA design is to limit ground multipath error. The elevation cutoff angle between two antennas is called the transition angle and is about 35 degrees.

To address ground reflection multipath, we begin with the following definitions: e is the ranging error due to ground reflection multipath, α is the amplitude of reflected signal relative to direct, θ is the phase of reflected signal relative to direct, d is the GPS receiver Delay Lock Loop (DLL) half correlator spacing (e.g., 0.05 chip or 15 m), and $\delta = 2h\sin E$ is the multipath delay. Neglecting second order and higher terms in α (since $\alpha^2 << 1$ for the MLA), a simplified error model for MLA ground multipath was defined in reference [2] as follows: $e = \min[\delta, d]\alpha\cos\theta$.

In the ground reflection multipath error model, the first term, $\min[\delta, d]$, is associated with antenna height, satellite position, and receiver correlator spacing. Since these values are known for any given site and satellite position, they may be defined as a constant value $c = \min[2h\sin E, d]$. Dividing ground reflection multipath by c results in the following normalized ground reflection multipath (NMP) error $\hat{e} = e/c = \alpha\cos\theta$.

In principle, NMP is a deterministic error source. However, it is impossible to fully characterize the physical/spatial characteristics and temporal variation in the local RF environment. Therefore, it is treated as a random error in this analysis. In the NMP error model, there are two variables: 1) relative signal strength, α, and 2) relative phase, θ. These two quantities will be treated as random variables. Candidate models for their distributions are described below. The goal is to define a worst-case, realistic distribution for these quantities for use in the overbounding analysis.

Relative Phase Variation: A uniform phase distribution (θ varies randomly between 0 and 2π) is a widely used model distribution in RF applications. Such a model is also consistent with the assumption that the phase varies over widely spaced days between 0 and 2π due to small changes in reflection surface height and reflectivity for a given satellite elevation. Thus, θ is first treated as a uniformly distributed random variable: $\theta \sim U(0, 2\pi)$. The corresponding PDF is: $f_\theta(\theta) = [u(\theta) - u(\theta - 2\pi)]\frac{1}{2\pi}$, where $u(.)$ is the unit step function. A second model for the phase, namely, a constant value of phase (worst-case scenario), is also considered. In making such an assumption, the effect of phase variation is directly eliminated since a constant value of phase is passed through the cosine function. In this case, for example, the relative phase distribution can be expressed by $f_\theta(\theta) = \delta_\theta(\theta - 2\pi)$. Where δ_θ is the Dirac Delta function.

Since we have defined candidate PDFs for relative phase variation, now, a new random variable, $z = \cos\theta$, can now be introduced. With a uniform phase variation, $\theta \sim U(0,2\pi)$, the corresponding PDF of z becomes, $f_z(z) = [u(1) - u(z-1)]\frac{1}{\pi\sqrt{1-z^2}}$. This PDF is symmetric, bi-modal, and has singularities at $z = \pm 1$. Bi-modality and singularity are important characteristics that lead difficulties in bounding process since the position domain bounding condition requires multiple convolutions of such PDFs. Therefore, special care is taken when using this non-gaussian ranging error distribution for bounding. With a constant phase, the PDF becomes $f_z(z) = \delta_z(z-1)$. In this case, the relative signal strength (α) distribution will be the only effective random source in the final PDF (as the PDF of a product of two independent random variables, α and z).

Relative Signal Strength: A worst-case constant value, b, is assumed for reflection amplitude, α. The PDF is expressed as $f_\alpha(\alpha) = \delta(\alpha - b)$. An upper limit for this constant value can be defined from the performance of MLA used in LTP as plotted in the upper trace of Figure 1 as a function of satellite elevation.

NON-GAUSSIAN AND NON-ZERO MEAN MULTIPATH ERROR DISTRIBUTIONS

The NMP error is a product of α and z. Since two distributions are defined for random variable z and a single distribution is defined for α, it is possible to generate two model distributions for NMP error as products of α and z. These models represent two limit case distributions of ground reflection multipath error: 1) *a worst-case symmetric zero-mean non-gaussian distribution* (i.e., a bi-modal distribution) and 2) *a mean value* (i.e., a bias-offset).

Additional less conservative models for ground reflection multipath were explored in [8], but the usefulness of these models is contingent on their experimental validation.

Model-1: Constant value of α and Uniformly distributed θ. With this model, the NMP error will be simply a product of a constant value of relative signal strength ($\alpha = b$, see Figure 1 for b values) and the random variable, z. The resulting PDF of NMP error is $f_{\hat{e}}(x) = [u(x-b) - u(x+b)]\frac{1}{\pi b\sqrt{1-(x/b)^2}}$. Direct analytical convolution of these PDFs yields integrals that are not tractable in closed form. The characteristic function (Fourier Transform) of the Model 1 PDF can be readily shown to be a Bessel Function of the first kind. However, the inverse Fourier Transform of products (equivalent to convolution in the range domain) of Bessel Functions is not readily accessible in closed form. Furthermore, direct numerical convolution of PDFs is also difficult since the PDF function has singularities at $\pm b$, requiring impractically fine discretization for accurate results. To circumvent these difficulties, we introduce a conservative approximation for this PDF as: $f_{\hat{e}}(x) = \frac{1}{2}[\delta(x-b) + \delta(x+b)]$. The associated CDF of NMP error is then, $F_{\hat{e}}(x) = \frac{1}{2}[u(x) - u(x-b)] + u(x)$.

We first consider the hypothetical limiting case of a convolution of a large number of Independent, Identically Distributed (IID) sources. In this case, the Central Limit Theorem defines the necessary bounding value of sigma for the actual PDF $\sigma \geq \sigma_{\hat{e}} = b/\sqrt{2}$ and for the conservative model $\sigma \geq \sigma_{\hat{e}} = b$. Unfortunately, while this result holds when N is very large, it is not sufficient for finite values of ($N \leq 12$). In this case, further inflation of the gaussian σ will be necessary to ensure CDF overbounding. For a given satellite n, we consider the NMP error, $\hat{e}_n$, is distributed according to the model $\hat{e}_n \sim \frac{1}{2}[\delta(x-\beta_n) + \delta(x+\beta_n)]$.

Since $\sigma_n = \beta_n$ for this distribution, the maximum possible NMP error resulting from a linear combination of the N sources is $\hat{e}_{max} \equiv \max\sum_{n=1}^{N}\hat{e}_n = \sum_{i=1}^{N}\beta_n = \sum_{n=1}^{N}\sigma_n \leq \sqrt{N}\sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_N^2} \equiv \sqrt{N}\sigma_{tot}$. Equality (largest e_{max}) in the bound above occurs when all β_n are the same (i.e., N IID sources). In this case, no inflation of σ_n is required when $\sqrt{N} \leq 2.878$ ($N \leq 8$) since the error will never exceed $2.878\sigma_{tot}$ (which is the limit criterion for position domain tail overbounding set by the definition of Category 1 LAAS VPL_{H1} in [1]). In general, an inflation/deflation factor of $\sqrt{N}/2.878$ may be used in the range domain to ensure that the position domain NMP error never exceeds $2.878\sigma_{tot}$. For example, for $N = 12$, an inflation factor of 1.204 is implied. However, a zero probability of exceeding $2.878\sigma_{tot}$ is clearly not necessary. We only require that the gaussian bounds the actual NMP error in the CDF sense above $2.878\sigma_{tot}$. A sufficient inflation factor of 1.05 is obtained from direct convolution of N (IID) sources. This result is sufficient for all N up to 12 (and conservative for $N < 9$).

We must also explicitly consider the effect of the addition of other contributing gaussian sources (due to diffuse MP and receiver noise). Clearly, the addition of such errors is not an issue in the following limiting cases:

- Gaussian sources are very small ($\sigma_{gauss} << \sigma_{\hat{e}}$): Model-1 dominates, so an inflation factor of 1.05 is sufficient.
- Gaussian sources are very large ($\sigma_{gauss} >> \sigma_{\hat{e}}$): Model-1 errors are negligible by comparison (Model-1 inflation factor is irrelevant).

In cases where gaussian and Model-1 errors are of comparable size, the results are again evaluated by direct convolution. In all cases, the inflation factor of 1.05 is seen to be sufficient to ensure overbounding. Finally, the ground reflection multipath sigma can be written as:

$$\sigma_e \geq 1.05bc \tag{12}$$

Model-2: Constant value of α and θ. With this model, the NMP error will be simply a product of a constant worst-case value of relative signal strength ($\alpha = b$, see upper trace of Figure 1) and a constant worst-case value of random variable, z. The normalized multipath error PDF for this case can be expressed as $f_{\hat{e}}(x) = \delta(x-b)$ and the associated CDF as, $F_{\hat{e}} = [u(x) - u(x-b)]$. With this PDF, the ground reflection multipath error is simply a bias value such as worst directions and maximum values (i.e., normalized ground reflection multipath error $\mu_{\hat{e}} \geq b$):

$$\mu_e \geq bc \tag{13}$$

As previously discussed in Model 1, we also explicitly consider the effect of the addition of other contributing gaussian sources (due to diffuse MP and receiver noise). Clearly, the addition of such errors results non-zero mean gaussian distributions. The existence of a non-zero mean in the correction error can result in unacceptable integrity risk. Therefore, the mean value must either be specifically accommodated via sigma establishment or it must be shown to have a negligible effect on integrity. Both of these two alternatives are pursued in following sections of paper.

Size of bc. The lower trace of Figure 1 shows the size of product bc versus satellite elevation for three different antenna heights. The upper trace shows the relative signal strength, $b = \alpha$. The relative signal strength values are obtained from [6] where they are suggested as minimum requirements for MLA antennas for LAAS.

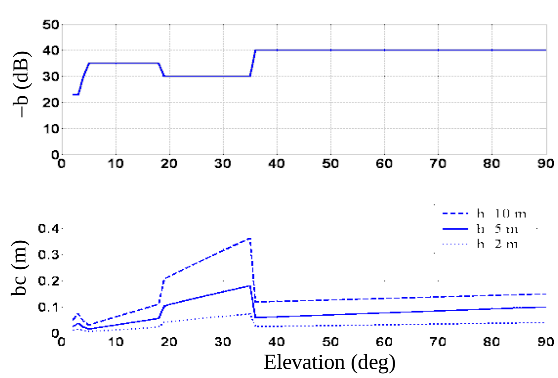

Figure 1 Size of Product bc Versus Satellite Elevation Angle for Three Different Antenna Heights

GAUSSIAN ERROR SOURCES

In LAAS, it is realized that using data alone is not sufficient to establish broadcast sigma, particularly if the data commissioning is limited to a single day (or a few days). The basic reason for this is that, with limited data, the distribution tails cannot be determined empirically. Simply treating the data as gaussian is consistent with certain component error sources, such as diffuse multipath and receiver thermal noise. However, such treatment is not sufficient by itself, because it is known that non-gaussian sources (ground reflection multipath) are also present. Therefore, while the overbounding solution should clearly be based on observed data, it must also be augmented by theoretical bounding of ground-reflected multipath. In the discussion below, we briefly review the candidate data processing methodology of reference [9] and show how multipath bounds described in this paper can be incorporated to generate the overall broadcast ground sigma. However, data processing approaches other than those described in [9] are equally applicable within the empirical/theoretical fusion methodology described here.

Serial Correlation and Nonstationarity. For a given data set (e.g., one day's worth of commissioning data), the effects of these two properties on the sigma estimate have a reciprocal relation as bin size changes. The adaptive bin method suggested in [9] avoids this difficult tradeoff by choosing the bin size at each elevation (E) such that the computed standard deviation (inflated for uncertainty due to the finite sample size within the bin) is maximized. The empirical result for a given reference receiver m and elevation E is denoted as $\sigma^{s}_{m,E}$.

Correlation Between Reference Receivers. In the LAAS Protection Level computations, it is implicitly assumed that ranging errors are uncorrelated across ground reference receivers. In reality, however, it is possible that some measurable correlation exists. Furthermore, even if a negligibly small correlation coefficient is computed from a finite sample set, the statistical uncertainty in the estimate must also be accounted for. Such uncertainty is lessened, as one would naturally expect, as the sample size used to estimate correlation coefficient increases. To accommodate the effects of correlation, detailed empirical methodologies are presented in [3] and [9] to define a factor β_m by which sigma should be increased: $\sigma^{sc}_{m,E} = \beta_m \sigma^{s}_{m,E}$.

Seasonal Variations. LGF multipath ranging error can change over time due to environmental factors. For practical reasons, however, it is clearly desirable to commission a candidate LGF using a short time span of data (e.g., over few days). One potential solution is to use archived LAAS Test Prototype (LTP) data. In reference [9] a method is described for inflation due to slow seasonal variations based on: 1) a normalization of long-term LTP error data with nominal sigmas computed on a single day, and 2) sigma inflation by a factor γ_m derived from the maximum observed seasonal variation between normalized sigmas. The result for a given reference receiver m and elevation E is $\sigma^{sct}_{m,E} = \gamma_m \sigma^{sc}_{E}$. The use of LTP data in such a manner may be sufficient until sufficient site specific data is collected.

GENERATION OF BROADCAST SIGMA

Synthesis of Broadcast Sigma. Because neither empirical error data nor theoretical approaches alone are adequate, the final broadcast σ_{pr_gnd} will be a result of both.

I. *Sigma Estimated From Data*. Gaussian (or nearly gaussian) error sources can be quantified using data. However, as noted above, the gaussian sigma estimates must include inflation to accommodate the following effects:

  -Sample standard deviation uncertainties (α).

  -Correlation between receivers (β).

  -Long-term temporal variations (γ).

  Candidate methods for these purposes are described in detail in [9] and briefly summarized in the previous section.

II. *Sigma Generated From Theoretical Analyses* ($\sigma_{MP,E}$): Overbounding gaussian distributions must be defined to accommodate the non-gaussian and/or non-zero mean effects due to ground reflection multipath. This subject was detailed earlier in this paper. (Additional related material may also be found in reference [8].)

In this section we apply these methodologies to LTP empirical data.

I. Sigma Estimated From Data. For an arbitrarily selected satellite, divergence-free code-minus-carrier error traces for three LTP reference receivers (RRs) are plotted in Figure 2. Also shown in the plots are the sigma traces (solid curve) computed using the adaptive bin method of [9]. These traces implicitly account for data nonstationarity and inflation due to statistical uncertainty (α). It is observed that the sigma results for RR1 and RR2 are generally larger than that for RR3. We should also note that the worst sigma values are obtained near the transition elevation angles (vertical dashed lines) between the High Zenith Antenna (which tracks high elevation satellites > 35 degrees) and the MLA (a dipole antenna that tracks low elevation satellites, ≤ 35 degree).

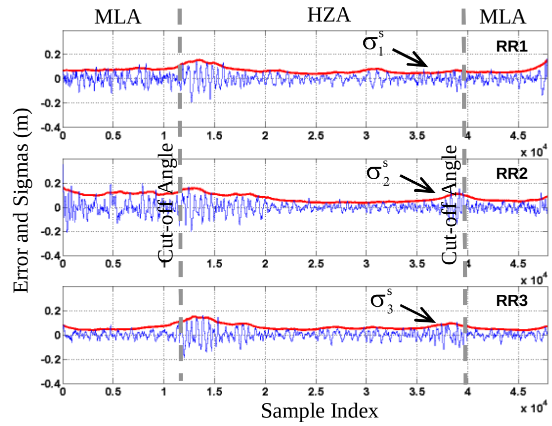

Figure 2 Adaptive Bin Sigma Results for Each RR

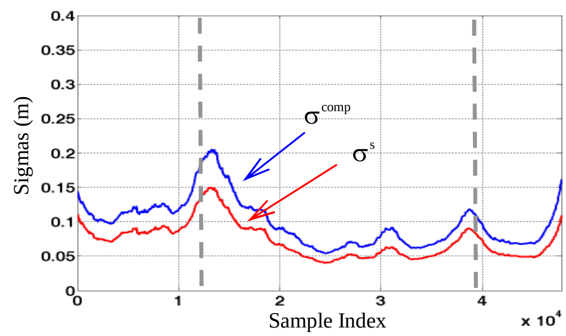

Figure 3 Composite Empirical Sigma Results

II. Sigma Generated From Theoretical Analyses

Candidate Broadcast Sigma for Model-1. Here we synthesize the empirical (composite) gaussian sigma result from Figure 3 with the gaussian overbound ground reflection multipath error sigma for Model 1 from Equation (12). The individual results are plotted in the upper and lower trace of Figure 4 respectively. As was shown earlier, the gaussian (empirical) and gaussian-multipath-overbound sigmas may be combined via root-sum-square to generate the final result.

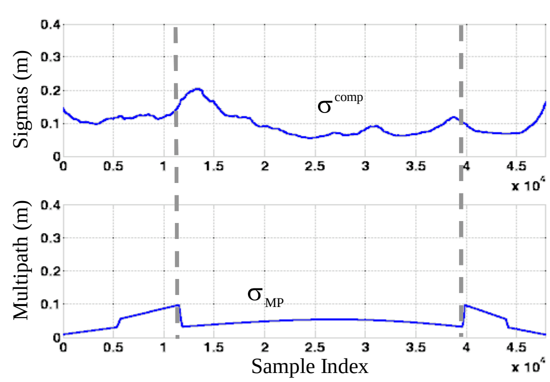

Figure 4 Multipath Model 1

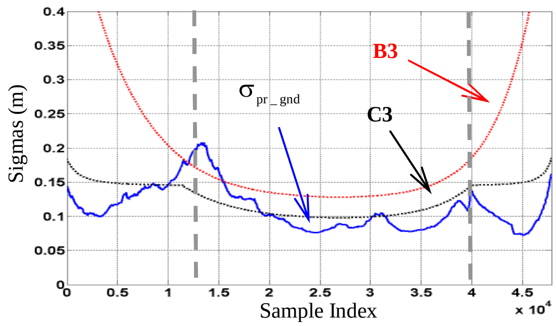

Figure 5 Combined Empirical and Model 1 Results

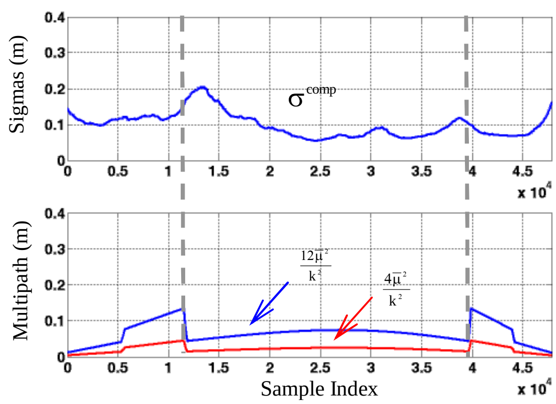

Figure 6 Multipath Model 2

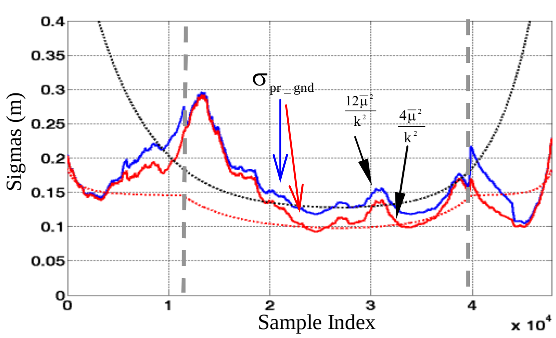

Figure 7 Combined Empirical and Model 2 Results

Candidate Broadcast Sigma for Model-2. Here the ground reflection multipath error is treated as worst-case bias (Model-2). In Figure 6, the upper trace is again the inflated empirical result from Figure 3, and the lower traces represent the bias contributions to the LGF bounding formula in Table 1. These two lower traces correspond to differing numbers of satellites in view at the LGF (the minimum, $N = 4$, and the maximum, $N = 12$).

The empirical and theoretical components plotted in Figure 6 are combined using the LGF mean bounding formula in Table 1 to generate the overall bounding sigmas in Figure 7.

It is clear that the results here are more conservative than the Model 1 case. This is an expected result because ground reflection multipath is treated as a worst case bias, with worst case phase, in Model 2, whereas in Model 1 credit is taken for randomness in the phase.

LAAS Availability Concerns. In these examples/results, navigation integrity is ensured for the established σ_{pr_gnd}. However, the sigma results exceed C3 and B3 specifications. This means that LAAS availability may be affected. Since both the empirical and theoretical contributions are relatively similar in magnitude, reductions in either component will have beneficial effects with regard to availability. In this regard, work is ongoing to refine both the empirical processes of [9] and the bounds developed in this paper to achieve tighter sigma bounds.

Negligibility of Mean Effects within the VAL. The existence of a non-zero mean value in the correction error can potentially result in unacceptable integrity risk. Therefore, either the mean value must be specifically accounted for (e.g., using one the methods defined above) or it must be shown to have a negligible effect on integrity. The second alternative was previously described in reference [8] and is briefly reviewed here because of its direct relevance to the subject at hand.

Given the computed VPL is defined in Equation (1), and the actual VPL given the existence of mean errors mean it is shown in reference [8] that a geometry-independent upper bound for the actual VPL may be defined as follows:

$$VPL_{act} \le \left(1 + \sqrt{N}\eta / k\right) VPL_{comp} \, . \tag{14}$$

$$\eta = \max_{n} \left| \mu_{pr\_gnd,n} / \sigma_{pr\_gnd,n} \right| . \tag{15}$$

In a practical interpretation of requirements it is possible that the 10 m VAL requirement is actually conservative for LAAS Category I operation. For example, while a conservative 'threshold' VAL of 10 m is used for LAAS, an 'operational' value of approximately 12 m may actually be sufficient to ensure integrity. In this regard, it is only necessary to ensure that the scaling term $\left(1 + \sqrt{N}\eta / k\right)$ is $12/10 = 1.2$ or lower. Using the smallest value of k for Category I (k_{md_ff} = 2.878 for H_1) and the largest number of satellites ($N_{max} = 12$), the maximum acceptable value of η is 0.167. The results for other values of N are shown in Figure 8. Recall that the number of tracked satellites is known to the LGF, so the least conservative value of η can always be used. Thus larger mean values can be accommodated as the number of satellites decreases. Note that this is a *sufficient condition* to ensure that the mean is negligible. Implementation may be achieved by verifying that the range-domain mean is small enough in comparison to the standard deviation as necessary to ensure that the sufficient condition is satisfied.

Since the condition defined above is sufficient but not strictly necessary (in particular, it can be conservative for good satellite geometries), alternatives are possible. For example, we may consider at the reference station (at any given time) all subset geometries that the aircraft may use to compute VPL . First assume the largest specification-compliant values of standard deviation for *air* and *residual* (tropospheric and ionospheric uncertainty) errors, and for all candidate subset geometries compute both VPL_{H_0} and VPL_{H_1}. The LGF verifies that in all cases where these VPLs are less than 10 m the addition of the unaccounted for position domain mean term does not cause the actual bounding VPL to exceed 12 m. Note that by itself, this is not truly a sufficient condition because the *air* and *residual* standard deviation values used at the aircraft may be smaller than the maximum values assumed on the ground. If they are smaller at the aircraft, it is possible that geometries exist in which both VPLs computed on the ground exceed 10 m (and thus are not verified) but at the aircraft are slightly less than 10 m. To protect against such occurrences, the ground station would also have to consider a *range* of standard deviation values for *air* and *residual* below the largest values assumed above. Such an approach would probably be computationally intensive, but is possible in principle.

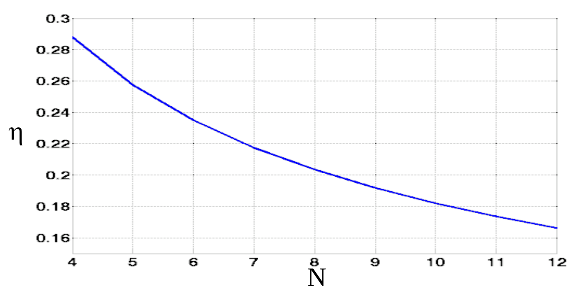

Figure 8 Maximum Allowable Ratios of Mean/Sigma

CONCLUSIONS

In this paper, two ground reflection multipath error distribution models were investigated for overbounding of LGF broadcast correction errors. Model-1 is a worst case, zero-mean, symmetric bi-modal distribution, and Model-2 is a worst-case bias. Model-2 is the most conservative interpretation of ground reflection multipath and it does not require any validation with data given that specified antenna/receiver design performance is achieved. This model is relevant to other systematic biases as well.

For both models, sufficient mathematical bounds on broadcast range domain σ_{pr_gnd} were defined to ensure error overbounding for the aircraft in the position domain. The conditions under which ground bias errors may be ignored in LAAS were also briefly reexamined

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REFERENCES

1. RTCA (SC-159/WG-4), "Minimum Aviation System Performance Standards for the Local Area Augmentation System (LAAS)," RTCA/DO-245, RTCA Inc., Washington DC, 28 September 1998.
2. Braasch, M., "Multipath Effects," *Global Positioning System: Theory and Applications, Volume 1*, AIAA Progress in Aeronautics and Astronautics Vol. 163,1996.
3. Pervan, B., and Sayim, I., "Sigma Inflation for the Local Area Augmentation of GPS," *IEEE Transactions on Aerospace and Electronic Systems*, Volume 37, Number 4, October 2001.
4. "Specification: Performance Type One Local Area Augmentation System Ground Facility," U.S. Federal Aviation Administration, Washington, D.C., FAA-E-2937, 21 September 1999.
5. Golub, H., Gene, and Loan, F. V. Charles, *Matrix Computations*, 2nd, Edition, The John Hopkins University Press, Maryland, 1993.
6. Braff, R., "Description of the FAA's Local Area Augmentation System of (LAAS)", *Navigation, Journal of The Institute of The Navigation*, Vol. 44, No. 4, Winter 1997-1998.
7. Enge, P., "Local Area Augmentation of GPS for the Precision Approach of Aircraft", *Proceedings of the IEEE*, Vol. 87, No.1, January 1999.
8. Pervan, P., Pullen, S., and Sayim, I., "Sigma Estimation, Inflation and Monitoring in the LAAS Ground System," *Proceedings of the 13th International Meeting of the Satellite Division of the Institute of Navigation* (ION GPS-2000), Salt Lake City, UT, September 2000.
9. Sayim, I., Pervan, B., Pullen, P., and, Enge, P., "Experimental and Theoretical Results on LAAS Sigma Overbound," *Proceedings of the 15th International Meeting of the Satellite Division of the Institute of Navigation* (ION GPS-2002), Portland, OR, September 2002.