CODE CARRIER DIVERGENCE MONITORING FOR THE GPS LOCAL AREA AUGMENTATION SYSTEM

BY

DWARAKANATH V SIMILI

DEPARTMENT OF MECHANICAL, MATERIALS AND AEROSPACE ENGINEERING

Submitted in partial fulfillment of the requirements for the degree of Master of Science in Mechanical and Aerospace Engineering in the Graduate College of the Illinois Institute of Technology

Approved _________________________

Adviser

Chicago, Illinois
May 2007
ACKNOWLEDGMENT

I take this opportunity to express my gratitude to a few people without whom this research would not have been possible. First and foremost, I’m indebted to my advisor, Dr. Boris Pervan for giving me the opportunity to pursue this research in the exciting field of satellite navigation. I would like to thank him for his willingness and patience in explaining new concepts and instilling a strong drive for analytical thinking in me. His support during my co-op term with Caterpillar Inc and guidance has been invaluable to me over the years as a graduate student, not only in matters related to research and course work but also in life as engineering professional. He has been a great source of inspiration and a true role model in more than one ways.

I would like to thank all my senior colleagues, Fang, Livio, Mattieu, Samer and Bart at NavLab for their useful inputs during the course of my research and in the presentations and discussions at the weekly NavLab meetings.

I sincerely appreciate the efforts of my thesis defense committee members and thesis examiner for making my defense possible on a short notice. The Federal Aviation Administration for sponsoring my research.

Finally, I would like to thank my father, S.V. Venkatesh, whom I have always looked up to, my mother Jayashree, brother Deepak, for all their love and support.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>ACKNOWLEDGEMENT</th>
<th>iii</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>vi</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>vii</td>
</tr>
<tr>
<td>LIST OF SYMBOLS</td>
<td>x</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>xiii</td>
</tr>
<tr>
<td>CHAPTER</td>
<td></td>
</tr>
<tr>
<td>1. I INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 The Global Positioning System</td>
<td>1</td>
</tr>
<tr>
<td>1.2 GPS Positioning</td>
<td>3</td>
</tr>
<tr>
<td>1.3 GPS Measurements</td>
<td>5</td>
</tr>
<tr>
<td>1.4 GPS Errors</td>
<td>7</td>
</tr>
<tr>
<td>1.5 Code Carrier Divergence</td>
<td>9</td>
</tr>
<tr>
<td>1.6 Differential GPS</td>
<td>12</td>
</tr>
<tr>
<td>1.7 Motivation</td>
<td>15</td>
</tr>
<tr>
<td>2. SYSTEM REQUIREMENTS</td>
<td>18</td>
</tr>
<tr>
<td>2.1 LGF Specification Requirements</td>
<td>19</td>
</tr>
<tr>
<td>2.2 MOPS Avionics Requirements</td>
<td>22</td>
</tr>
<tr>
<td>2.3 Aircraft Filters</td>
<td>25</td>
</tr>
<tr>
<td>3. MONITOR ALGORITHM</td>
<td>34</td>
</tr>
<tr>
<td>3.1 Divergence Rate Estimator</td>
<td>34</td>
</tr>
<tr>
<td>3.2 Fault Free Distribution</td>
<td>37</td>
</tr>
<tr>
<td>3.3 Filtered Code Noise Contribution</td>
<td>43</td>
</tr>
<tr>
<td>3.4 Monitor Threshold</td>
<td>46</td>
</tr>
<tr>
<td>4. MONITOR INTEGRITY ANALYSIS</td>
<td>47</td>
</tr>
<tr>
<td>4.1 Integrity Analysis Fundamentals</td>
<td>47</td>
</tr>
<tr>
<td>4.2 Integrity Analysis - CCD Monitoring</td>
<td>51</td>
</tr>
<tr>
<td>4.3 CCD Monitor</td>
<td>54</td>
</tr>
</tbody>
</table>
5. CONCLUSIONS........................................................................................................... 60

APPENDIX
A. HZA RESULTS ........................................................................................................... 64
B. MLA RESULTS .......................................................................................................... 69

BIBLIOGRAPHY ............................................................................................................. 74
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Nominal Ionosphere Divergence Data Archive</td>
<td>39</td>
</tr>
<tr>
<td>2</td>
<td>Nominal Ionosphere Divergence Rate</td>
<td>43</td>
</tr>
<tr>
<td>3</td>
<td>$\sigma_{dn}$ (Uninflected) vs. Elevation and Time Constant</td>
<td>44</td>
</tr>
<tr>
<td>4</td>
<td>$\sigma_{dn}$ vs. Elevation and Time Constant</td>
<td>44</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>1.1</td>
<td>Hierarchies of GPS Capabilities</td>
<td>14</td>
</tr>
<tr>
<td>1.2</td>
<td>Schematic Showing Mismatch in Filter Time Constants and Filter Start Times</td>
<td>16</td>
</tr>
<tr>
<td>2.1</td>
<td>MOPS Compliant Transient Response</td>
<td>25</td>
</tr>
<tr>
<td>2.2</td>
<td>1st Order LTI Filter Responses</td>
<td>26</td>
</tr>
<tr>
<td>2.3</td>
<td>1st Order LTI Filter: Noise Response</td>
<td>27</td>
</tr>
<tr>
<td>2.4</td>
<td>1st Order LTI Filter: Divergence Response</td>
<td>28</td>
</tr>
<tr>
<td>2.5</td>
<td>1st Order LTV Filter: Noise Response</td>
<td>29</td>
</tr>
<tr>
<td>2.6</td>
<td>1st Order LTV Filter: Divergence Response</td>
<td>30</td>
</tr>
<tr>
<td>2.7</td>
<td>2nd Order LTI Filter: Noise Response</td>
<td>31</td>
</tr>
<tr>
<td>2.8</td>
<td>2nd Order LTI Filter: Divergence Response</td>
<td>31</td>
</tr>
<tr>
<td>2.9</td>
<td>2nd Order LTV Filter: Noise Response</td>
<td>32</td>
</tr>
<tr>
<td>2.10</td>
<td>2nd Order LTV Filter: Divergence Response</td>
<td>33</td>
</tr>
<tr>
<td>3.1</td>
<td>Divergence Rate Estimator Model</td>
<td>35</td>
</tr>
<tr>
<td>3.2</td>
<td>1st Order Filter (τ_{d1} = 200 sec)</td>
<td>36</td>
</tr>
<tr>
<td>3.3</td>
<td>Two 1st Order Filters in Series (τ_{d1} = τ_{d2} = 40 sec)</td>
<td>36</td>
</tr>
<tr>
<td>3.4</td>
<td>Example Divergence Trace: SV 3 Feb 11 2004</td>
<td>39</td>
</tr>
<tr>
<td>3.5</td>
<td>CDF Plot for Combined LTP Divergence Data</td>
<td>40</td>
</tr>
<tr>
<td>3.6</td>
<td>Gaussian Overbound of LTP Ionospheric Divergence Rate Data</td>
<td>41</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.7</td>
<td>Cumulative Distribution Function of Normalized Ionosphere Temporal Gradients on July, 2000</td>
<td>42</td>
</tr>
<tr>
<td>3.8</td>
<td>Normalized CDF Plot of Divergence Estimate Error for HZA</td>
<td>45</td>
</tr>
<tr>
<td>3.9</td>
<td>Normalized CDF Plot of Divergence Estimate Error for MLA</td>
<td>45</td>
</tr>
<tr>
<td>4.1</td>
<td>LGF Integrity Risk Given Fault on Ranging Source $k$</td>
<td>50</td>
</tr>
<tr>
<td>4.2</td>
<td>Ground and Air Filter Responses (Relative to Steady State) for Divergence Rate input $d_{nom} = 0.018$ m/sec.</td>
<td>53</td>
</tr>
<tr>
<td>4.3</td>
<td>Divergence Rate Estimator Response to Unit Ramp</td>
<td>54</td>
</tr>
<tr>
<td>4.4</td>
<td>Three-Dimensional Contour Plots of $P_{LOI</td>
<td>\text{fault}}$: Category I Case</td>
</tr>
<tr>
<td>4.5</td>
<td>Illustration for $P_{(ev</td>
<td>\text{fault})}$ and $P_{md</td>
</tr>
<tr>
<td>4.6</td>
<td>Worst-case $P_{LOI</td>
<td>\text{fault}}$ for any value of $t$, $t_{0g}$, and $t_{0a}$ versus Magnitude of Divergence Rate Failure: Category I Case</td>
</tr>
<tr>
<td>A.1</td>
<td>Example HZA Divergence Estimate</td>
<td>65</td>
</tr>
<tr>
<td>A.2</td>
<td>HZA Divergence Estimate Error vs Time Constant</td>
<td>65</td>
</tr>
<tr>
<td>A.3</td>
<td>Example Elevation vs Time</td>
<td>66</td>
</tr>
<tr>
<td>A.4</td>
<td>Example $C/N_o$ vs. Time</td>
<td>66</td>
</tr>
<tr>
<td>A.5</td>
<td>$1st$ Order LTI Filter: Noise Response</td>
<td>67</td>
</tr>
<tr>
<td>A.6</td>
<td>Divergence Estimate Error vs Elevation for 3 SVs: 13, 28, and 31</td>
<td>67</td>
</tr>
<tr>
<td>A.7</td>
<td>Divergence Estimate Error vs $C/N_o$ for 3 SVs: 13, 28, and 31</td>
<td>68</td>
</tr>
<tr>
<td>B.1</td>
<td>Example MLA Divergence Estimate</td>
<td>70</td>
</tr>
<tr>
<td>B.2</td>
<td>MLA Divergence Estimate Error vs Time Constant</td>
<td>70</td>
</tr>
<tr>
<td>B.3</td>
<td>Example $C/N_o$ vs Time</td>
<td>71</td>
</tr>
</tbody>
</table>
LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.4</td>
<td>Example Elevation vs. Time</td>
<td>71</td>
</tr>
<tr>
<td>B.5</td>
<td>C/N₀ vs. Elevation for 3 SVs: 13, 28, and 31</td>
<td>72</td>
</tr>
<tr>
<td>B.6</td>
<td>Divergence Estimate Error vs. Elevation for 3 SVs: 13, 28, and 31</td>
<td>72</td>
</tr>
<tr>
<td>B.7</td>
<td>Divergence Estimate Error vs. C/N₀ for 3 SVs: 13, 28, and 31</td>
<td>73</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>Filter time constant, equal to 100 seconds</td>
</tr>
<tr>
<td>$T$</td>
<td>Filter sample interval, nominally equal to 0.5 seconds</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Accumulated phase measurement</td>
</tr>
<tr>
<td>$k$</td>
<td>Current measurement</td>
</tr>
<tr>
<td>$k-1$</td>
<td>Previous measurement</td>
</tr>
<tr>
<td>$\tau_{d1}, \tau_{d2}$</td>
<td>Monitor filter time constants</td>
</tr>
<tr>
<td>$e_v$</td>
<td>Vertical component of differential position error due to all sources</td>
</tr>
<tr>
<td>$v_i$</td>
<td>Differential ranging error for satellite $i$ due to all fault-free error sources</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>Standard deviation of $v_i$</td>
</tr>
<tr>
<td>$b_k$</td>
<td>Differential ranging error on satellite $k$ due to satellite failure only</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of satellites in view</td>
</tr>
<tr>
<td>$S$</td>
<td>Weighted pseudo inverse used at aircraft for position fix (which is a function of satellite geometry)</td>
</tr>
<tr>
<td>$S_{v,i}$</td>
<td>The element of matrix $S$ projecting the ranging measurement from satellite $i$ into the vertical direction</td>
</tr>
<tr>
<td>$PR_r$</td>
<td>Raw pseudorange</td>
</tr>
<tr>
<td>$PR_s$</td>
<td>The smoothed pseudorange</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_k$</td>
<td>Test statistic for the fault monitor for satellite $k$ due to the satellite failure only</td>
</tr>
<tr>
<td>$\eta_k$</td>
<td>Test statistic for the fault monitor for satellite $k$ due to all fault-free error sources</td>
</tr>
<tr>
<td>$\sigma_{\eta,k}$</td>
<td>Standard deviation of the monitor test statistic in the fault-free case</td>
</tr>
<tr>
<td>$\sigma_{r,k}$</td>
<td>Standard deviation of the monitor test statistic for faulted case</td>
</tr>
<tr>
<td>$\sigma_{pr_gnd}$</td>
<td>Standard deviation of the error on the broadcast smoothed pseudorange correction (LGF)</td>
</tr>
<tr>
<td>$\sigma_{pr_gnd,nom}$</td>
<td>Standard deviation of ground ranging error due to all sources except filter transient to nominal ionospheric divergence</td>
</tr>
<tr>
<td>$\sigma_{div_gnd}$</td>
<td>Standard deviation that accounts for ground transient filter responses to nominal ionospheric divergence</td>
</tr>
<tr>
<td>$\sigma_{pr_air}$</td>
<td>Standard deviation of the error on the broadcast smoothed pseudorange correction (aircraft)</td>
</tr>
<tr>
<td>$\sigma_{pr_air,nom}$</td>
<td>Standard deviation of air ranging error due to all sources except filter transient to nominal ionospheric divergence</td>
</tr>
<tr>
<td>$q_k$</td>
<td>Test statistic for the fault monitor for satellite $k$ due to all sources</td>
</tr>
<tr>
<td>$\sigma_{div_air}$</td>
<td>Standard deviation that accounts for air transient filter responses to nominal ionospheric divergence</td>
</tr>
</tbody>
</table>
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>DEFINITION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_{ffd,mon}$</td>
<td>The multiplier on $\sigma_{\eta,k}$ used to define monitor threshold with desired fault free detection probability</td>
</tr>
<tr>
<td>$Q$</td>
<td>Cumulative distribution function for the standard normal distribution</td>
</tr>
<tr>
<td>$\sum_{i=1}^{n}S_{v,i}v_i$</td>
<td>Vertical component of differential position error for all fault-free error sources</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Standard deviation of $\sum_{i=1}^{n}S_{v,i}v_i$</td>
</tr>
<tr>
<td>$k_{ffmd}$</td>
<td>5.81, the multiplier on $\sigma_r$ used to compute LAAS $VPL_{H0}$</td>
</tr>
</tbody>
</table>
ABSTRACT

Code-carrier smoothing is a commonly used method in Differential GPS (DGPS) systems to mitigate the effects of receiver noise and multipath. The Federal Aviation Administration’s (FAA) Local Area Augmentation System (LAAS) uses this technique to help provide the navigation performance needed for aircraft precision approach and landing. However, unless the reference and user smoothing filter implementations are carefully matched, divergence between the code and carrier ranging measurements will cause differential ranging errors.

The FAA’s LAAS Ground Facility (LGF) reference station will implement a prescribed first-order Linear Time Invariant (LTI) filter. Yet flexibility must be provided to avionics manufacturer’s in their in airborne filter implementations. While the LGF LTI filter is one possible means for airborne use, it’s relatively slow transient response (appropriate for a ground based receiver) is not ideal at the aircraft because of frequent filter resets following losses of low elevation satellite signals (caused by aircraft attitude motion). However, in the presence of a Code-Carrier Divergence (CCD) failure at the GPS satellite, arbitrarily large divergence rates are theoretically possible, and therefore protection must be provided by the LGF through direct monitoring for such events. In response, this research addresses the impact of the CCD threat to LAAS differential ranging error and defines a LGF monitor to ensure navigation integrity.

Differential ranging errors resulting from unmatched filter designs and different ground/air filter start times were analyzed in detail, and the requirements for the LGF CCD monitor were derived. A CCD integrity monitor algorithm was then developed to directly estimate and detect anomalous divergence rates. The monitor algorithm was
implemented and successfully tested using archived field data from the LAAS Test Prototype (LTP) at the William J. Hughes FAA Technical Center.
CHAPTER 1
INTRODUCTION

With different technologies present today to determine ones position on earth the Global Positioning System (GPS), which is a radio based satellite navigation system, is one of the most successful methods with ever growing list of applications. It was developed by the Department of Defense (DoD) in 1970s primarily for U.S. military to provide precise estimates of position, velocity and time. Ever since, over the years with system development, civil applications of GPS have grown at an astonishing rate.

The first chapter of this dissertation presents the fundamental concepts of the GPS architecture and then introduces the need for Differential GPS (DGPS) and its implementation by Federal Aviation Administration (FAA): the Local Area Augmentation System (LAAS) program for precise aircraft landing. This chapter also presents the phenomenon of Code Carrier Divergence (CCD) and explains the need for CCD monitoring at the LAAS ground facility (LGF), which is the motivation for this research.

1.1 The Global Positioning System

GPS consists of three segments the space segment, control segment and the user segment. The space segment of GPS comprises of a baseline constellation of 24 satellites in nearly circular orbits with 12 hour periods implying they pass over the same location in sky twice a day and a radius of approximately 20,000 km. There are four satellites placed unevenly in six orbit planes that have inclinations of 55 degrees. The system can support a constellation of up to thirty satellites on orbit. With the current setup almost all users with a clear view of the sky have a minimum of four satellites in view, ensuring
GPS availability 24 hours a day anywhere on earth. The current GPS satellites are called the Block IIF with DoD in the process of acquiring new generation satellites called the GPS III aided with extensive modification to Block IIF.

The control segment is a network of monitoring stations placed strategically over the Earth, with the Master Control Station (MCS) located at Colorado Springs, Colorado. This station operates the system and provides the command and control functions, such as monitoring and correcting satellite orbits, satellite health maintenance, maintaining GPS time, and updating broadcast satellite navigation messages. The other monitor stations are unmanned and operated remotely from the MCS.

Finally, the User Segment is the user’s GPS receiver which is responsible for the large scale use of GPS in civil applications as these receivers are compact, light and using relatively less expensive clocks (as opposed to atomic clocks used on board the satellites). Low-end receivers are available off the shelf for $100, while more sophisticated models (with high quality clocks and antennas) are priced anywhere around $10,000 and over. These receivers help determine the location of the user by measuring the distance from the satellite to the user; these distances are computed by measuring the time it takes for the radio signal broadcast from the satellite to reach the receiver.

With high dependability of space and control segments aided with small size and cost of receivers, many practical applications of GPS exist. The main application considered in this research is to navigate aircraft in complete automatic landing or navigate the aircraft up to Category I decision height using GPS. The proceeding sections will describe GPS positioning, errors associated with GPS, two main types of GPS
measurements and how these measurements are combined to implement Differential GPS (DGPS) for LAAS.

1.2 GPS Positioning

As mentioned we can determine position on earth by measuring the distance from various satellites. GPS has a minimum of 24 satellites in its constellation. A user would need at least four range measurements from these satellites to compute a position fix. Measurements from all satellites are not required and also not possible (due to visibility constraints). Out of the four measurements, three are needed to give position information, i.e. the x, y, z coordinates of the user if the unknown user’s location is expressed in Earth Centered Earth Fixed (ECEF) coordinates (Cartesian coordinates with origin at the center of the earth). A fourth measurement is needed to resolve the receiver clock bias. The distance from the satellite is determined by measuring the transit time of the signal being broadcast. In order to accurately measure this transit time the receiver clock (less precise and low cost) needs to be synchronized with the satellite atomic clock (one of the most expensive components on board a GPS satellite). As this is clearly not a practical and feasible option the receiver clock bias is treated as a unknown and the fourth measurement is required to determine the user location. Hence we have four or more measurements and four unknowns so that the user position and clock offset can be calculated.

Next, we focus our attention to the type of signal that the satellite continuously broadcasts that is recognizable by the user receiver. The GPS satellite transmits two radio frequencies in the L- band referred to as Link-1(L1) and Link-2(L2). The L- band covers
frequencies between 1 GHz and 2 GHz, and is a subset of the Ultra High frequency (UHF) band. The center frequencies of L1, L2 are 1575.42 MHz and 1227.60 MHz.

The satellite broadcasts navigation data at 50bps and a Coarse Acquisition (C/A) code at 1.023 MHz both modulated on L1 and a precise (P) code at 10.23 MHz along with navigation data modulated on L2. When the GPS signal is received the navigation data and the C/A code are demodulated, with the navigation data containing information of the ephemerides of the satellite orbits among other information. From the ephemerides the satellite position can be computed at any given point. The distance from the now known satellite position to the user location is found using the C/A code. The C/A code is input into a closed tracking loop called the Delay Lock Loop (DLL). In the receiver an internally generated replica of the C/A code is compared with C/A code being broadcast by the satellites. Note, each satellite has its own unique C/A code. When the C/A code is received the DLL in the receiver will phase shift its replica until the correlation between the two signals is maximized. The phase corresponds to the transit time of the signal to reach the user receiver, and range is computed by multiplying with speed of light.

As mentioned earlier, in order for this method to work accurately the clock that is used to generate the signal and the clock that is used to track the signal must be synchronized. The GPS satellites have very accurate clocks that are closely synchronized with each other, but on the other hand the receiver clocks for example a quartz oscillator which is relatively less expensive, or a temperature compensated crystal oscillators [van95] can also be used. For the purpose of navigation low end receivers do the job this makes GPS a very viable option for day to day civil users in applications ranging from land survey, trekkers having hand held GPS units to determine their position, automobiles
and even highly advanced applications like automatic landing of aircrafts and unmanned aerial vehicles. In the next section we present the basic forms of GPS measurement signals and their formulation to account for different errors which are inherent from the time the signal leaves the satellite till it reaches the user receiver.

1.3 GPS Measurements

**Code Phase Measurement:** This is the most basic form of measurement made by a GPS receiver. It is defined as the difference between signal reception time, as determined by the receiver clock, and the transmission time at the satellite, as marked on the signal. This measurement is the amount of time shift required to align the C/A code replica generated at the receiver with the signal received from the satellite. As the satellite and receiver clocks keep time independently they are not synchronized and hence this measurement is biased in nature. Therefore the biased range or pseudorange is defined in terms of the transit time measured multiplied by the speed of light in vacuum. The equation representing code phase measurement accounting for the clock bias and delays associated with transmission of the signal through the ionosphere ($I_\rho$) and troposphere ($T_\rho$) is given by,

$$\rho(t) = r(t) + c[\delta t_u(t) - \delta t'(t - \tau)] + I_\rho(t) + T_\rho(t) + \varepsilon_\rho(t)$$

(1-1)

Where, $\tau$ is the transit time associated with a specific code transition from the satellite, $\rho$ is the pseudo range measurement at epoch $t$, $c$ speed of light, $\delta t'(t - \tau)$ is the transmission time stamped on the signal and $\delta t_u$ being the arrival time measured by the receiver clock.
Carrier Phase Measurement: This form of measurement is much more precise than that of code phase measurement. The carrier phase measurement is the difference between the phases of the receiver generated carrier signal and the corresponding carrier signal received from the satellite at the instant of measurement. This measurement is indirect and ambiguous of the signal transit time.

This is can be explained as follows, in a idealized world with error free measurements with perfectly synchronized satellite and receiver clocks, and no relative motion between the user and satellite, the carrier phase measurement would remain fixed at a fraction of cycle plus the distance between the satellite and receiver with unknown number of whole cycles, referred to as integer ambiguity. Now when the carrier phase is tracked and there is relative motion between the user and satellite the distance between them grows by wavelength. To measure carrier phase in GPS the receiver acquires phase lock and keeps track of the change in measurement, counting the full carrier cycles and the initial fractional cycle at each epoch. The basic equation representing this measurement with errors is given by,

\[
\phi(t) = \lambda^{-1}[r + I_\phi + T_\phi] + \frac{c}{\lambda}(\delta t_u - \delta t') + N + \epsilon_\phi
\]  

(1-2)

where, \( \lambda \) is the carrier wavelength, \( N \) is the integer ambiguity, \( I_\phi \) and \( T_\phi \) are the ionosphere and troposphere associated propagation delays in meters, note the carrier phase measurements in Equation 1-2 is in units of cycles.

As seen from Equations (1-1) and (1-2) both code and carrier phase measurements are corrupted by the same error sources, the important difference being code measurements are unambiguous and coarse by comparison. The carrier measurements are
precise but contain integer ambiguities. The integers remain constant as long as the phase lock is maintained, but the loss of lock, no matter how short could change the integer values.

In order to take full advantage of these measurements to obtain accurate position estimates we have to resolve the integer ambiguities and compensate for various errors. The errors associated with the code and carrier phase measurements are discussed in detail in the next section.

1.4 GPS Errors

The errors associated with GPS measurements can be grouped into errors due to space and control segment, the signal propagation medium and the user receiver.

The control segment is responsible for errors in parameter values broadcast by the satellite in its navigation message, for example Satellite Clock and Satellite Ephemeris errors.

The Ephemeris error is composed of error components along the three orthogonal directions defined relative to the satellite orbit; radial, cross track and along track. The range error due to the errors in clock and ephemeris parameters is generally characterized terms of root sum square (rms) value. The size of this error is estimated and tracked by the control segment in real time with in about 1m rms.

The next sets of errors are associated with the propagation medium, which affect the travel time of the signal from the satellite to the receiver, for example ionospheric delay and Tropospheric delay. When the signal propagates through the ionosphere the modulation is delayed while the phase of the carrier is advanced. These phenomena are
known as group delay and phase advance respectively. The delay due to the ionosphere can be as high as 25 meters at the zenith. This error can vary depending on obliquity or slant factor.

The second atmospheric error is due to the Troposphere. This layer is present below the ionosphere, its primary constituents being water vapor and dry gases. This medium is non dispersive (refractive index is constant with frequency) at L1 and L2 frequencies so the carrier is delayed by the same amount as the code. The contribution to ranging error due to Troposphere can vary between 2 to 25 meters [Spi94] but this error can be modeled more accurately than the ionosphere, hence such large errors are never seen.

The last set of uncertainties associated with GPS measurement is due to receiver noise that affects the precision of measurement. The random noise which in a broader sense covers the RF radiation sensed by the antenna in the band of interest contains noise introduced by the antenna, amplifiers and cables hence, the receiver cannot follow the changes in the signal waveform perfectly and invariably there are delays and distortions. The measurement error due to receiver noise varies with GPS signal strength which in turn is related to satellite elevation angle. With existing receiver technology it is not difficult to attain a measurement precision of 0.5%-1% of a cycle, which translates to about 3meters error on code and 0.2 centimeters on carrier measurements( in the absence of other non-noise errors).

The interference from signals reflected from the surfaces in the vicinity of the antenna is referred to as Multipath. As a result of multipath, the code and carrier phase measurements are corrupted and the error in range measurements depends on the strength
of the reflected signals and the delay between reflected and direct signals. Multipath affects both code and carrier measurements, but the magnitudes of the errors are significantly different. Using typical antennas and receivers, a code phase measurement error varies from 0.5 meter to more than 3 meters depending on the environment and for carrier phase it is of the order 1-3 centimeters.

The rms value of the above mentioned errors can lead to a typical user range error of 5 to 10 meters. This corresponds to the range between the user and a particular satellite at a given instant of time. In order to obtain the user actual position the ranging accuracy must be multiplied by the Position Dilution of Precision (PDOP). The PDOP is a value that depends on the geometry of constellation of satellites used at the time of measurement. The PDOP can vary anywhere from two to as high as six or seven [Park94]. Keeping this in mind the attained position accuracy is sufficient for a biker/tracker wanting to determine his position on a map, but for today’s evolving advanced technologies like autonomous ground vehicles, aircraft, unmanned aerial vehicles (UAV’s), the GPS accuracy must be further improved.

1.5 Code Carrier Divergence

As mentioned in the earlier section GPS signals are affected by the medium through which they travel from the satellite to the receiver. The earth’s atmosphere changes both direction and speed of radio signals and this changes the signal transit time, which is the basic measurement for GPS. When the GPS signals travel through the ionosphere the measurement of carrier phase is too short and the modulation signal (i.e. code and navigation message) measured is too long by an equal amount. This phenomenon is referred to as Code-Carrier Divergence (CCD). This is as a result of
phase advance and group delay [Misr01] of radio waves as they travel through ionized
gases in a dispersive medium and is in direct proportion to Total Electronic Content
(TEC) along the propagation path of a signal. The path of a signal through the ionosphere
depends upon the elevation of the satellite and is accounted for by the obliquity factor
[Misr01].

**Combining Code and Carrier Phase Measurements:** In this section we briefly
discusses the benefits of combining the noisy but unambiguous code phase measurements
with precise but ambiguous carrier phase measurements for absolute positioning in a
single receiver autonomous mode.

It is generally convenient to write [Misr01] carrier phase measurement in units of
length like code phase measurements and for simplicity in order to explain this method
we rewrite the measurement equations as,

\[
\rho^* = r(t) + c[\delta t_0 (t) - \delta t' (t - \tau)] + T(t)
\]  \hspace{1cm} (1-3)

where, \( \rho^* \) is the ionosphere free measurement and we rewrite code and carrier phase
equations as,

\[
\rho(t) = \rho^*(t) + I(t) + \varepsilon_\rho (t)
\]

\[
\phi(t) = \rho^*(t) - I(t) + \lambda N + \varepsilon_\phi (t)
\]  \hspace{1cm} (1-4)

As mentioned earlier the carrier phase measurement is ambiguous hence we set
the starting value at time \( t_0 \) arbitrarily to zero. Then the change in code and carrier phase
measurements between two measurement epochs \( t_{i-1} \) and \( t_i \) can be written as,

\[
\Delta \rho(t_i) = \rho(t_i) - \rho(t_{i-1}) = \Delta \rho^*(t_i) + \Delta I(t_i) + \Delta \varepsilon_\rho (t_i)
\]

\[
\Delta \phi(t_i) = \phi(t_i) - \phi(t_{i-1}) = \Delta \rho^*(t_i) - \Delta I(t_i) + \Delta \varepsilon_\phi (t_i)
\]  \hspace{1cm} (1-5)
where, \( \Delta \rho^* \) is, the change in ionosphere free pseudorange measurement between the two measurement epochs, similarly \( \Delta I \) and \( \Delta \varepsilon \) correspond to changes in ionospheric delay and error term.

Now, we have two unknowns in two equations. The noise in the first term is at meter level, and in the second term it is at centimeter level. There are two possible approaches for this situation. One is that we can appropriately weight the two equations and solve for \( \Delta \rho^* \) and \( \Delta I \) terms. The second approach is to discard \( \Delta I \) term, assuming that it is small when the two measurement epochs are close together, and use \( \Delta \phi(t_i) \) as an accurate estimate of \( \Delta \rho^* \).

The estimate of \( \rho(t_0) \) is available from each epoch as,

\[
\hat{\rho}(t_0) = \rho(t) - [\phi(t) - \phi(t_0)]
\]

We can average these estimates over \( n \) epochs and obtain an estimate to reconstruct the smoothed pseudorange profile given by,

\[
\tilde{\rho}(t_i) = \hat{\rho}(t_0) + [\phi(t_i) - \phi(t_0)]
\]

Carrier smoothing of code is now a regular feature in modern receivers to help mitigate the effect of receiver code noise and multipath. An efficient implementation of the above idea is by using a recursive filter of length \( N \) given by,

\[
\begin{align*}
\tilde{\rho}(t_i) &= \frac{1}{N} \rho(t_i) + \frac{(N-1)}{N} \left[ \rho(t_i-1) + (\phi(t_i) - \phi(t_i-1)) \right] \\
\tilde{\rho}(t_i) &= \rho(t_i)
\end{align*}
\] (1-6)
The filter weighs the carrier phase measurements more heavily than code phase measurements. Remember, however, that in deriving the above equation we neglected the change in ionosphere between two measurement epochs. While this change is negligible over few seconds one must be careful that this does not add up to be significant over N epochs, because the impact is doubled as the change effects the two measurements in opposite directions.

1.6 Differential GPS

Positioning using a single GPS receiver is done by code based pseudoranges, perhaps smoothed by carrier phase. But this is insufficient if the user wants to very high (e.g.1m level) position accuracy. In order to achieve higher accuracy we must reduce the measurement errors discussed in the earlier section. This requires a change in the mode of GPS usage from single receiver autonomous positioning to Differential GPS (DGPS).

The fundamental idea behind DGPS is to take advantage of the fact that the satellites are so far away (20,000 Km) as compared to the distance between the receivers (terms of few km for aircraft precision approach) the errors associated with GPS measurements like satellite clock, ephemeris and the atmospheric propagation are similar and these errors change slowly with time. This is because these errors exhibit temporal and spatial correlations. Since these errors are common using DGPS we can cancel the common errors between the receivers leaving only the non-common errors to deal with. the non-common errors being multipath and receiver errors which are environment and receiver specific.

DGPS accomplishes this by employing a reference station and a dynamic user receiver. The location of the reference station is known accurately while the dynamic user
is free to move, keeping in mind that the receivers must be in relatively close proximity. The purpose of the reference station is to compute the error in pseudorange measurement form a particular satellite and send the corrections over a communication link to the dynamic user. The dynamic user then uses these corrections to determine its accurate position from the same satellite. The reference station is able to do this, as it accurately knows its own position and the position of the satellite from the navigation message received from the satellite. Comparing the computed range with the actual ranging measurement at the reference station gives the ranging error for that particular satellite.

Figure 1.1 [Eng99] shows potential accuracies of GPS and DGPS. There are two types of DGPS presented Code DGPS and Carrier DGPS. The later form of correction is more accurate but requires resolving cycle integer ambiguity [Misr01]. This type of DGPS is used for surveying but is usually not a practical solution for real time navigation applications. In code DGPS, the measurements are more accurate than regular GPS but still have errors due to the influence of code multipath and receiver noise. These errors by themselves provide a hurdle to use raw code DGPS for our application which is precision landing of aircraft using Local Area Augmentation System (LAAS).
LAAS is being developed by the FAA and the aviation industry to serve all three categories (CAT I, II, III) of precision approach. LAAS is a local area differential GPS system because it places all of its reference receivers close together with in a few hundred meters and forms a single correction for each satellite to be used by incoming aircraft. A of high quality GPS reference receivers are located at surveyed locations at the airport. The reference measurements (or corresponding corrections) are broadcast to approaching aircraft using a VHF data link.

LAAS augments GPS navigation in two ways. First, the LAAS Ground Facility (LGF) provides differential corrections to the user (aircraft) that augment user accuracy. Second, the LGF monitors the ranging sources to protect against faults that could result in navigation errors, thereby ensuring the integrity of navigation for LAAS users.
In order to ensure system integrity, for each class of ranging source (i.e., satellite) fault there is a monitor at the LGF that computes a test metric designed to indicate the presence of that particular type of fault. The purpose of these monitors is to limit the LAAS user’s integrity risk due to a fault occurring at the ranging source.

1.7 Motivation

Keeping the above introduction in mind we next present the motivation behind this research and provide a well-defined problem statement.

In this dissertation we address the code-carrier divergence (CCD) fault. Divergence can be caused by ionospheric activity (nominal or storm) or a fault occurring at the ranging source (satellite). The latter is the primary CCD threat, as large divergence rates are theoretically possible. This motivates the need for CCD monitoring to be provided by the LGF. During abnormal ionospheric activity, the CCD monitor can also provide benefit by helping to detect moving storm fronts.

Like other DGPS architectures, the LGF (reference station) and aircraft (user) employ code-carrier smoothing to mitigate the effects of receiver noise and multipath. If the aircraft and ground implement identical filter designs and have the same start times, then they experience the same transient response to divergence, and differential-ranging errors will not exist. However, as shown in Figure 1.2 if they have unmatched filter designs or different start times, then divergence between code and carrier ranging measurements will cause differential ranging errors.

The LGF reference station will implement an FAA prescribed first-order Linear Time Invariant (LTI) filter. One approach to mitigate the CCD threat is to have the
ground and aircraft implement identical smoothing filters. The cost, however, is a loss of design flexibility for avionics manufacturers, which is not desirable. Even though the LGF LTI filter is one possible means for airborne use, its relatively slow transient response (acceptable for a ground based receiver) is not ideal at the aircraft because the aircraft filter is likely to experience frequent filter resets following the loss of signals from low elevation satellites (caused by aircraft attitude motion). In this dissertation we will consider various possible aircraft filter implementations and show that a first order Linear Time Varying (LTV) filter is a good choice for airborne implementation.

![Image](image.png)

**Figure 1.2 Schematic Showing Mismatch in Filter Time Constants and Filter Start Times**

In this work we develop, implement and experimentally validate a new CCD monitor algorithm. The CCD monitor algorithm developed has two functions: divergence rate estimation and detection. Monitor thresholds are established with the aid of LAAS.
Test Prototype (LTP) data. We present a new direct approach for integrity analysis and prove that the monitor meets the integrity risk requirement for category I aircraft precision approach. This analysis also provides a framework for treating in general space segment failures on satellites that cause a different transient response in the differential position error.
CHAPTER 2
SYSTEM REQUIREMENTS

This chapter presents the specifications for the CCD monitor to be implemented at the LGF. The smoothing filter to be implemented at the LGF (reference station) and the aircraft (user) are also discussed. The requirements for the development of the monitor are based on meeting the integrity requirements for LAAS program. The specifications for the implementation of smoothing filter at the aircraft give sufficient room for the avionics manufacture to design different order filters which would give different transient response to a CCD failure. A preliminary analysis is carried out for selecting best possible smoothing filter that can be implemented at the aircraft.

Before we define the system requirements a few standard definitions [Eng99] with respect to GPS navigation are given below as these lay the foundation upon which the requirements are written for the LAAS program.

Accuracy is the difference between the estimated and true aircraft position. More specifically, navigation sensor error (NSE) is defined as the difference between the estimated and true position. NSE is the error that is only exceeded 5% of the time in the absence of system failures.

Integrity and Continuity characterize the system response to component failures or rare natural events- they are the main measure of flight safety. They address the ability of a navigation system to detect and possibly repair system threats in a timely fashion. Integrity fails when the position error exceeds a certain Alert Limit (AL, measured in meters). This error is not annunciated to the pilot or the aircraft guidance system within a specified time to alert. Integrity risk is defined as the probability that all the alerts are
silent while the aircrafts position error exceeds the specified AL. For precision approaches these alert limits are given in terms of Vertical Alert Limit (VAL), Horizontal Alert Limit (HAL) and time to alarm.

On the other hand continuity fails when an aircraft operation must be aborted for any unscheduled reason. Continuity risk is defined as the probability that the navigation system will fail during a flight operation given that it was available at the beginning of the operation. Basically, continuity and integrity are competing requirements, because continuity requires the total number of missed approaches to be very small, requiring that integrity algorithms not be overly sensitive. If they are unduly sensitive it would result in too many false alarms being sent to the aircraft, causing unnecessary missed approaches will be conducted. Keeping these basic definitions in mind, the system requirements are given in the section below.

2.1 LGF Specification Requirements

This section summarizes the relevant LGF specifications and requirements on ranging source integrity, ground monitor continuity, and the prescribed LGF smoothing filter algorithm. The relevant Minimal Operational Performance Standards (MOPS) avionics requirements are also discussed. Further details on requirements can be found in [LGF02, MOP01].

2.1.1 LGF Category I Integrity Requirement. The probability that the LGF transmits Misleading Information (MI) for 3 seconds or longer due to a Ranging Source (RS) failure shall not exceed $1.5 \times 10^{-7}$ during any 150 sec approach interval.
a) MI is defined as broadcast data that results in the lateral or vertical position error exceeding protection levels for any user w/in 60 nmi of the LGF.

b) The CCD failure rate is defined to be $10^{-4}$/hr for satellite (SV) acquisition, and the prior probability of CCD failure after acquisition is given as $4.2 \times 10^{-6}$/SV/approach.

2.1.2. LGF Reference Receiver (RR) and Ground Monitor Continuity Requirement. It is required that the probability of any valid ranging source is made unavailable due to a false alarm shall not exceed $2.3 \times 10^{-6}$ per 15 sec interval.

For the purposes of this work, we assume that the relevant allocations (from items 2.1.1 and 2.1.2 above) for the CCD monitor are:

- The probability of MI given a CCD fault is $10^{-4}$ per 150 sec approach interval, and the probability of fault free alarm is $10^{-7}$ per 15 sec interval. The reduction in the MI probability form that defined in 2.1.1 results from the fact that $10^{-4}$ is a conditional probability given that the fault actually exists. The assumed prior probability of fault is effectively $1.5 \times 10^{-3}$ per 15 sec interval.

2.1.3 LGF Smoothing Filter Algorithm. The smoothing filter is defined in the LGF specification as follows [LGF02]. “In steady state, each pseudorange measurement from each RS shall be smoothed using the filter:

\[
PR_s(k) = \left( \frac{1}{N} \right) PR_s(k) + \left( \frac{N-1}{N} \right) \left[ PR_s(k-1) + \phi(k) - \phi(k-1) \right]
\]

\[N = \frac{S}{T}\]

The raw pseudorange is derived under the following conditions:
a) The code loop shall be carrier driven and of first order, or higher, and have a one-sided noise bandwidth $\geq 0.125$ Hz.

b) The strongest correlation peak is acquired.”

Notes: The one-sided noise bandwidth requirement is equivalent to a 2 sec upper limit on the Delay Lock Loop (DLL) filter time constant. In the LGF, the DLL time constant is 0.5 sec. LGF will also implement the filter above without modification during filter initialization as well as steady-state.

The LGF is also required to generate and broadcast to LAAS user $\sigma_{pr\_gnd}$, the standard deviation of the error on the broadcast smoothed pseudorange correction. The variable $\sigma_{pr\_gnd}$ can be expressed as the root sum square (RSS) of the standard deviation of ground ranging error due to all sources except filter transient to nominal ionospheric divergence ($\sigma_{pr\_gnd, nom}$) and the standard deviation that accounts for ground transient filter responses to nominal ionospheric divergence ($\sigma_{div\_gnd}$).

The nominal CCD rate is given as normally distributed with zero mean and standard deviation of 0.018 m/s. After filter startup (or reset) the LGF must inflate $\sigma_{pr\_gnd}$ from its nominal steady-state value ($\sigma_{pr\_gnd, nom}$) to account for the transient deviation from the steady state response to this nominal input divergence rate:

$$\sigma_{pr\_gnd} = RSS(\sigma_{pr\_gnd, nom}, \sigma_{div\_gnd})$$

where, $\sigma_{div\_gnd} = 0.018 \cdot (100.5) \cdot e^{-\frac{(t-t_{0g})}{100}}$ (in meters), $t_{0g}$ is the LGF filter start time (sec), and $t$ is the current time (sec). Based on the LGF specification presented in the above section for this research we considered the following requirements [ADD05]:

2.1.4 Category I Integrity Risk Due To Undetected CCD Failures. Of the total RS integrity risk of 1.5×10⁻⁷/approach allowed by LGF, as allocated in [ADD05] we used 1.96×10⁻⁸ to undetected CCD failures. Using the CCD failure rate defined in LGF specification, and assuming:

a) 18 ranging sources in operation;

b) An average of 0.2 ranging sources acquired during any 150-second approach exposure time, and

c) 6-hour exposure time for period during which satellite is not in view.

From [ADD05], we used the following prior probabilities for the CCD threat:

- Operational: 18 SVs × 4.2×10⁻⁶/SV/approach = 7.56×10⁻⁵/approach
- Acquisition: 6 hr × 1.0×10⁻⁴/SV/hr = 6.0×10⁻⁴/SV

Given a CCD failure, the allocated conditional probability \( P_a \) of out of tolerance information—referred to as “missed detection” is related to the above allocations as follows: \( P_a \times (7.56\times10^{-5}/\text{appr} + 6.0\times10^{-4}/\text{SV}\times0.2\ \text{SV/appr}) = 1.96\times10^{-8}/\text{appr} \). Therefore, for Category I \( P_a = 10^{-4} \) for any 150 sec approach interval. For continuity fault tree, of the total RR and ground monitor continuity risk of 2.3×10⁻⁶ per 15 sec interval allowed by LGF, the allocated probability of fault free alarm for the CCD monitor function is \( P(FA) = 10^{-7} \) per 15 sec interval.

2.2 MOPS Avionics Requirements

This section gives the MOPS requirements [MOP01] for the aircraft smoothing filter and the possible realizable MOPS-compliant aircraft filters (discussed in later sections) that can potentially provide benefit to the aircraft. We investigate the worst-case
differential filter response for different combinations of LGF and aircraft filter start times.

The airborne system is required to do carrier smoothing of pseudorange (code) measurements, but a specific filter is not defined. (This differs from the LGF specification, which prescribes the ground system filter implementation.) Instead the MOPS provides significant flexibility to avionics manufacturers by specifying that the airborne filter need only match the ground filter with the following performance requirement:

“In response to a code-carrier divergence rate of up to 0.018 m/s, the smoothing filter output shall achieve an error less than 0.25 m within 200 sec after initialization relative to the steady-state response of [filters specified in [LGF02]].”

The LGF filter is one acceptable means for airborne use, but it is not the only possible implementation. Furthermore there is no explicit DLL bandwidth requirement for avionics in MOPS, just that the reference filter (i.e., the LGF filter described above) uses raw pseudorange from a “code loop carrier driven, 1st order or higher and with a one-sided noise bandwidth greater than or equal to 0.125Hz.” [LGF02]

MOPS also states that, $\sigma_{pr\_gnd}$ must account for filter transient response to nominal divergence. After filter startup (or reset) the aircraft must inflate $\sigma_{pr\_air}$ to account for the transient deviation “relative to the steady state response of the filter [i.e., LGF filter], given code carrier divergence of 0.018 m/s.” [MOP01]. It is also specified that the steady state value of RSS ($\sigma_{\text{noise}}, \sigma_{\text{div\_air}}$) for GPS satellites shall be:

- At minimum GPS signal level RSS ($\sigma_{\text{noise}}, \sigma_{\text{div\_air}}$) $\leq 0.15$ m
• At maximum GPS signal level RSS \( (\sigma_{\text{noise}}, \sigma_{\text{div \_air}}) \leq 0.11 \text{ m} \)

Because the avionics must handle both high and low signal levels and divergence response is not a function of signal level, it is required that:

• At steady state \( \sigma_{\text{div \_air}} \leq 0.15 \text{ m} \)

Therefore, the LGF knows that \( \sigma_{\text{div \_air}} \) is less than 0.15 m at steady state.

The MOPS states that the aircraft must account for its filter transient response to nominal divergence (i.e., after filter startup or reset) by inflating the standard deviation used for ranging measurements in the derivation of its protection levels. This effect is captured by \( \sigma_{\text{div \_air}} \). The MOPS further requires that the steady state value of \( \sigma_{\text{div \_air}} \) shall not exceed 0.15 m, and it states that “steady state operation is defined to be following 360 seconds of continuous operation of the smoothing filter.”[MOP01].

The relevant MOPS requirements can be summarized in graphical form as shown in Figure 2.1. The aircraft is permitted to have transient response to divergence of 0.018 m/s anywhere in the unshaded region. The response of the LGF filter (one acceptable choice at the aircraft) is shown below,
This section describes realizable MOPS-compliant aircraft filters that can potentially provide benefit to the aircraft and also investigates the worst-case differential filter response for different combinations of LGF and aircraft filter start times.

Figure 2.2 compares example 1st order LTI avionics filter time responses with different time constants (for an input divergence rate of 0.018 m/s) to the MOPS requirements. The minimum and maximum LTI filter time constants to ensure MOPS compliance are 100 sec and 108.8 sec, respectively.
Figure 2.2 1st Order LTI Filter Responses

When the order and time-invariance of the avionics filter are unconstrained, there exists even more flexibility in transient response to divergence inputs. Unfortunately, any significant variation from the LGF filter response is problematic because it will be realized as a differential ranging error. The ground filter implementation is known to the avionics manufacturer (it is defined in the LGF Specification), so there is little practical incentive to deviate from the LGF response. The aircraft must account for any such deviation in $\sigma_{\text{div.,air}}$, thereby increasing $\sigma_{\text{pr.,air}}$ and reducing system availability. To maximize system availability, it is reasonable to assume that the goal of the avionics manufacturer is to keep $\sigma_{\text{pr.,air}}$ small at any given time. For divergence inputs, this means that minimizing overshoot and steady-state error will lower $\sigma_{\text{pr.,air}}$. For nominal code noise inputs, time varying filter implementations (at filter start-up) can provide quicker reduction in $\sigma_{\text{noise}}$, and therefore $\sigma_{\text{pr.,air}}$. In this section, we show practical first and second order avionics filter designs that are potentially useful. Higher order filter designs
are possible, but they are unlikely to be employed because they do not reduce $\sigma_{pr,\text{air}}$ relative to direct airborne use of the LGF filter and are also more complicated to implement.

2.3.1 First Order LTI Filter. Figures 2.3 through 2.4 show the transient responses of 1\textsuperscript{st} order LTI and Linear Time Varying (LTV) MOPS-compliant aircraft filter implementations. Figure 2.1.1 shows the sample response to a white code noise input ($\sigma = 0.5$ m) for the LGF LTI filter along with the theoretical 1-sigma error envelope.

Figure 2.4 shows the response of the same filter to a divergence input of 0.018 m/s. Recall that this filter is too implemented at LGF but is merely one acceptable means at aircraft. The slow transient response of this filter to noise will affect the aircraft more seriously than the ground because filter resets will happen more frequently at the aircraft than at the LGF. This is true because the nominal attitude motion of the aircraft will cause intermittent losses of signal lock for low-elevation satellites.

![Figure 2.3 1st Order LTI Filter: Noise Response](image)

Figure 2.3 1st Order LTI Filter: Noise Response
2.3.2 First Order LTV Filter. Using a time varying gain in the first order filter at the aircraft can significantly speed up the transient response to noise. Figure 2.5 shows a sample noise response using a filter with time constant $\tau = t$ for $t < 100$ sec and $\tau = 100$ sec for $t \geq 100$ s. The rapid convergence of the error is readily apparent relative to the LTI case. Furthermore, the filter is simple to implement in that it deviates from the LGF LTI filter only in that filter gains are changing in the first 100 sec.

The response of the LTV filter to a divergence input of 0.018 m/s, shown in Figure 2.6, suggests that the MOPS divergence response requirements are not met. However, when the MOPS definition of steady state (360 sec) is used to define the lower transient response boundary, the time response becomes acceptable. From the point of view of LGF CCD monitor design; this filter must therefore be treated as a serious candidate for airborne implementation.
Figure 2.5 1st Order LTV Filter: Noise Response

Figure 2.6 1st Order TV Filter: Divergence Response
2.3.3 Second Order Filters. The use of second order filters at the aircraft can also be MOPS compliant. Figures 2.7 and 2.8 show the noise and divergence responses of a second order LTI filter which was designed to simultaneously maximize overshoot (subject to the MOPS divergence response requirements) and yet produce negligible steady state error relative to the LGF implementation.

Figure 2.7 2nd Order LTI Filter: Noise Response

As with the first order case, LTV realizations of the 2\textsuperscript{nd} order filter are also possible to speed up noise response. Figures 2.9 and 2.10 show responses from an example implementation where the time constant of the 2\textsuperscript{nd} order LTI filter above changes linearly with time (as in the 1\textsuperscript{st} order LTV case). While there is no clear time response benefit in the use of a 2\textsuperscript{nd} order filter in the avionics relative to the LTV 1\textsuperscript{st} order filter, the noise output of the 2\textsuperscript{nd} order filter does have less high-frequency content. This
may be a beneficial feature to a navigation avionics manufacturer who desires to produce a smoother position inputs to the autopilot.

Figure 2.8 2nd Order LTI Filter: Divergence Response

Figure 2.9 2nd Order LTV Filter: Noise Response
The above section under the current MOPS allows for wide variability in filter implementations at the aircraft. In the process of the development and analysis of the CCD monitor, it became clear that it was not possible to guarantee that all MOPS compliant aircraft would be protected against CCD failures. For further analysis in this dissertation we recommend a 1st order LTI or LTV (100 sec time constant in steady state) implementation for the aircraft avionics. In the next chapter we present the CCD monitor algorithm development that is the second phase in this research work. The chapter gives simulation results from real world data to help determine the monitor time constants and show the effect of ionosphere on divergence rates.
CHAPTER 3

MONITOR ALGORITHM

Category I LAAS ranging source monitors protect against satellite faults that could result in differential errors between the aircraft and the ground systems by evaluating test matrices and excluding a satellite when one of its corresponding matrices exceeds a certain threshold.

The CCD monitor is to be implemented at the LGF. One must keep in mind that this monitor can also help detect moving ionosphere fronts, but the LGF has other monitors dedicated to detect the anomalies of the ionosphere which could result in a CCD fault.

The LGF divergence monitor consists of two components: a divergence rate estimator and a detection test. A separate monitor function is implemented for each ranging source being tracked by the LGF. The input, $z(k)$, to the monitor at each 2 Hz time epoch $k$ is the raw code minus carrier

$$z(k) = PR_c(k) - \phi(k)$$

and the output is a one-bit flag, $p(k)$, indicating detection or no detection.

3.1 Divergence Rate Estimator

As shown in Figure 3.1, the divergence rate estimator differentiates the input $z$ and filters the result using two 1st order LTI filters in series to reduce the code noise contribution to estimate error. The estimator algorithm is given in equations (3-2) and (3-3) below. The estimator output is the filtered divergence estimate $d_2$.
\[ d_1(k) = \frac{\tau_{d1} - T}{\tau_{d1}} d_1(k-1) + \frac{1}{\tau_{d1}} [z(k) - z(k-1)] \]  \hspace{1cm} (3-2) \\

\[ d_2(k) = \frac{\tau_{d2} - T}{\tau_{d2}} d_2(k-1) + \frac{T}{\tau_{d2}} d_1(k) \]  \hspace{1cm} (3-3) 

The filter time constants \( \tau_{d1} \) and \( \tau_{d2} \) are algorithm parameters (modifiable), whose values are nominally set at \( \tau_{d1} = \tau_{d2} = 25 \) sec. The justification for these particular time constant values will be provided shortly. As illustrated in Figures 3.1, the use of two 1\textsuperscript{st} order filters in series (which is equivalent to a second order filter with real poles) reduces estimate error caused by differentiating high-frequency code measurement noise when compared to a single 1\textsuperscript{st} order filter. The effect is clearly demonstrated in Figures 3.2 and 3.3, which show filter outputs to simulated white noise with standard deviation of 0.5 m along with the theoretically derived standard deviation envelopes. It is clear in these two figures that the performance of the 2\textsuperscript{nd} order implementation with 25 sec time constant is superior (lower output noise) to a 1\textsuperscript{st} order implementation, even when the latter has a much longer filter time constant (200 sec). The ability to use a shorter time constant will result in quicker detection of CCD failures.

Figure 3.1 Divergence Rate Estimator Model
Following the divergence estimator is the detection function, which is a simple threshold test:

\[
\text{if } d_2 > T_{ccd}, \text{ then } p = 1 \quad /* \text{Alarm} */ , \\
\text{else } p = 0 \quad /* \text{No Alarm} */
\]
where, the detection threshold is defined as
\[ T_{ccd} = k_{fbd} \sigma_d, \]  
(3-4)

and \( p \) is a one-bit flag capturing the pass/fail result of the test. In equation (3-4), \( \sigma_d \) is the fault-free standard deviation of the test statistic \( d_2 \) and \( k_{fbd} \) is a constant chosen to ensure that the probability of fault-free alarm meets the allocated continuity requirement for the monitor. Both \( \sigma_d \) and \( k_{fbd} \) are algorithm parameters (modifiable), whose values are nominally set at \( \sigma_d = 0.004 \text{ m/s} \) and \( k_{fbd} = 5.73 \). The justification for these particular parameter values will be provided in the next sections.

3.2 Fault Free Distribution

As stated above, in order to set the monitor threshold, we need the fault-free standard deviation (\( \sigma_d \)) of test statistic \( d_2 \). This fault-free distribution will be affected by both filtered code noise (\( \sigma_{dn} \)) and nominal ionospheric divergence (\( \sigma_{di} \)):

\[ \sigma_d = \sqrt{\frac{\sigma_{dn}(EL, \tau_{d1}, \tau_{d2})^2 + \sigma_{di}^2}{\sigma_{di}}} \]  
(3-5)

where, \( EL \) is the satellite elevation. We will first consider the nominal ionospheric contribution.

3.2.1 Nominal Ionospheric Divergence Contribution. The nominal value for ionospheric divergence used in the LGF Specification and the MOPS (\( \sigma_{di} = 0.018 \text{ m/s} \)) was selected to ensure that the protection levels computed at the aircraft would be conservatively large. However, assuming such a large value for \( \sigma_{di} \) the CCD monitor will cause unreasonably loose monitor thresholds. In fact, prior research [Chr99] suggests that a more realistic nominal value is \( \sigma_{di} \approx 0.003 \text{ m/s} \). To corroborate this result
and define a useful value of $\sigma_{\text{di}}$ for the CCD monitor two sources of nominal ionospheric data were used:

a) Archived dual frequency carrier phase data from the LTP: This data was collected at a single site but included multiple normal-ionosphere days spanning nearly one year. The raw LTP data was supplied by the William J. Hughes FAA Technical Center.

b) WAAS “super truth” ionospheric data, which was processed using data from all WAAS sites for one normal ionospheric day. The data was processed at Stanford University to extract divergence rates.

As described below, a zero mean gaussian distribution with a standard deviation of $\sigma_{\text{di}} = 0.004$ m/s overbounds the cumulative distribution of the observed normal ionospheric divergence rates observed in the data.

**Ionospheric Divergence Analysis Using LTP Data.** Ashtech receiver L1/L2 carrier phase data (1 Hz) was used with a 5 deg elevation mask to compute instantaneous ionospheric divergence rates over 1 sec measurement intervals. The instantaneous rates were then averaged in 100 sec windows to reduce the differentiated carrier phase noise contribution to the rate estimates. The analysis was carried out for 17 satellites over 7 months with data taken from one day in each of these months (see Table 1 for details). Figure 3.4 shows an example divergence trace for a single satellite on a single pass.
Table 1. Nominal Ionosphere Divergence Data Archive

<table>
<thead>
<tr>
<th>SV No.</th>
<th>Date (Year 2004)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 20</td>
<td>Feb 11, Mar 11, Apr 16, Jun 15, Jul 15, Aug 31, and Oct 05</td>
</tr>
</tbody>
</table>

Figure 3.4 Example Divergence Trace: SV 3 Feb 11 2004
Figure 3.5 CDF Plot for Combined LTP Divergence Data

Figure 3.5 shows the cumulative distribution function (CDF) of all of the empirical divergence rates from the entire data set. The dashed line in the plot is the CDF of a gaussian distribution with the same standard deviation as the data ($\sigma_{di} = 0.0014$ m/s). It is clear that the ionospheric divergence rate distribution is not gaussian in the distribution tails. However, Figure 3.6 shows that when the gaussian standard deviation is inflated by a factor of 2.85, the Gaussian distribution does bound the empirical CDF. The nominal ionospheric divergence rates in the archived LTP data are therefore bounded by a gaussian distribution with $\sigma_{di} = 0.004$ m/s.
Ionospheric Divergence Analysis Using WAAS Supertruth Data. Wide Area Augmentation System (WAAS) post-processed network data (high-precision estimates of ionospheric delay known as “supertruth”) and a “Time-Step” method were used to compute ionospheric divergence rates for a nominal day. The ionosphere delay between the satellite and receiver at one epoch ($T_1$) was compared with the delay for the same pair at a later epoch ($T_2$). The differential delay was divided by the time interval, $\Delta t = T_1 - T_2$, to estimate the underlying ionospheric temporal gradient. The ionospheric divergence rates were then computed by multiplying these ionospheric temporal gradients by a factor of two. Observations from all visible satellites and 25 WAAS reference stations were used to estimate ionosphere divergence rate statistics.

Figure 3.7 shows the CDF of ionospheric temporal gradients (in the slant domain) from data collected on July 2, 2000. A time step of $\Delta t = 60$ seconds was used to

---

* This analysis was provided by Sam Pullen, Jiyun Lee, and Youngshin Park of Stanford University.
generate the data in this figure. Again, it is clear that the actual distribution (the dotted curve), derived from the empirical data, has non-Gaussian tails. However, a Gaussian distribution does overbound the empirical data when an inflation factor of 1.58 on the standard deviation is used (solid curve).

Use of other time step values leads to similar results as shown in Table 2. However, time steps of $\Delta t = 30$ seconds and shorter tend to magnify the effects of the underlying noise in the data. The table shows one-sigma estimates of the ionosphere temporal gradient $\sigma_{tg}$, the required inflation factors, $\sigma_{tg}$ overbounds, and $\sigma_{di}$ overbounds (two times the $\sigma_{tg}$ overbounds). The divergence rates on this nominal ionospheric day are bounded by a Gaussian distribution with $\sigma_{di} = 0.0038$ m/s. This result is very similar to the LTP data analysis result above.

![Figure 3.7 Cumulative Distribution Function of Normalized Ionosphere Temporal Gradients on July 2, 2000](image)

Figure 3.7 Cumulative Distribution Function of Normalized Ionosphere Temporal Gradients on July 2, 2000
Table 2. Nominal Ionosphere Divergence Rate

<table>
<thead>
<tr>
<th>Date</th>
<th>Time Interval (sec)</th>
<th>( \sigma_{tg} ) (m/s)</th>
<th>Inflation Factor</th>
<th>( \sigma_{tg} ) Overbound</th>
<th>( \sigma_{di} ) Overbound</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 2, 2000</td>
<td>200</td>
<td>0.0011</td>
<td>1.60</td>
<td>0.0018</td>
<td>0.0036</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>0.0011</td>
<td>1.60</td>
<td>0.0018</td>
<td>0.0036</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>0.0011</td>
<td>1.58</td>
<td>0.0018</td>
<td>0.0036</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.0012</td>
<td>1.64</td>
<td>0.0019</td>
<td>0.0038</td>
</tr>
</tbody>
</table>

3.3 Filtered Code Noise Contribution

As indicated in equation (3-5), the contribution of filtered code noise (\( \sigma_{dn} \)) depends on EL and the filter time constants \( \tau_{d1} \) and \( \tau_{d2} \). In principle, \( \tau_{d1} \) and \( \tau_{d2} \) can always be chosen large enough to make \( \sigma_d \approx \sigma_{di} \) (although care must be taken to ensure that the monitor detection time does not become too large). For simplicity in the monitor implementation and analysis, we define \( \tau_d = \tau_{d1} = \tau_{d2} \).

 Archived field test data was used to determine \( \sigma_{dn} \) as a function of EL for different values of \( \tau_d \). Data from LGF receivers with Multipath Limiting Antennas (MLA) and High Zenith Antennas (HZA) were used as the source of code and carrier data for input into the divergence rate estimator in equations (3-2) and (3-3). To isolate the code noise contribution, carrier phase data from a nearby NovAtel OEM4 receiver was used to remove nominal ionospheric divergence from the data prior to processing. Intermediate results for the processed data are given in Appendix A and B. Table 3 provides a summary of the raw empirical results for \( \sigma_{dn} \) in m/sec (sample standard deviation of elevation-binned data).
Figures 3.8 and 3.9 show the CDF of the divergence estimator error for HZA and MLA data using filters with $\tau_d = 25$ sec. The CDF data in these figures are normalized by the raw empirical values of $\sigma_{dn}$ in the second row (i.e., $\tau_d = 25$ sec) in Table 3. The results show Gaussian overbounding of the empirical data is ensured when inflation factors of 1.75 and 2.2 are applied to the empirical values of $\sigma_{dn}$ for the HZA and MLA, respectively. The inflated results for $\sigma_{dn}$ are shown in Table 4. Relative to $\sigma_{d}$ (0.004 m/sec), is clear from table that the resulting $\sigma_{dn}$ is an order of magnitude smaller. The contribution of $\sigma_{dn}$ to $\sigma_{d}$ is therefore negligible (in the RSS) for $\tau_d = 25$ sec. Monitor threshold using this time constant is defined in the section below.

### Table 3 $\sigma_{dn}$ (Uninflated) vs. Elevation and Time Constant

<table>
<thead>
<tr>
<th>$\tau_d$</th>
<th>EL</th>
<th>5-15° (MLA)</th>
<th>15-30° (MLA)</th>
<th>30-40° (HZA)</th>
<th>40-50° (HZA)</th>
<th>50-60° (HZA)</th>
<th>&gt;60° (HZA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 sec</td>
<td></td>
<td>0.000577</td>
<td>0.00060</td>
<td>0.000519</td>
<td>0.000299</td>
<td>0.00023</td>
<td>0.00021</td>
</tr>
<tr>
<td>25 sec</td>
<td><strong>0.00024</strong></td>
<td><strong>0.00022</strong></td>
<td><strong>0.00023</strong></td>
<td><strong>0.000148</strong></td>
<td><strong>0.000088</strong></td>
<td><strong>0.000085</strong></td>
<td></td>
</tr>
<tr>
<td>40 sec</td>
<td></td>
<td>0.000166</td>
<td>0.000165</td>
<td>0.000137</td>
<td>0.00008</td>
<td>0.000062</td>
<td>0.000057</td>
</tr>
<tr>
<td>60 sec</td>
<td></td>
<td>0.000106</td>
<td>0.000082</td>
<td>0.000062</td>
<td>0.000037</td>
<td>0.000029</td>
<td>0.000028</td>
</tr>
</tbody>
</table>

### Table 4 $\sigma_{dn}$ vs. Elevation and Time Constant

<table>
<thead>
<tr>
<th>$\tau_d$</th>
<th>EL</th>
<th>5-15° (MLA)</th>
<th>15-30° (MLA)</th>
<th>30-40° (HZA)</th>
<th>40-50° (HZA)</th>
<th>50-60° (HZA)</th>
<th>&gt;60° (HZA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 sec</td>
<td></td>
<td>0.0007509</td>
<td>0.000632</td>
<td>0.000713</td>
<td>0.000398</td>
<td>0.0002416</td>
<td>0.00024</td>
</tr>
<tr>
<td>25 sec</td>
<td></td>
<td>0.000529</td>
<td>0.000487</td>
<td>0.000517</td>
<td>0.0002603</td>
<td>0.000155</td>
<td>0.00015</td>
</tr>
<tr>
<td>40 sec</td>
<td></td>
<td>0.000191</td>
<td>0.0001891</td>
<td>0.00018</td>
<td>0.000109</td>
<td>0.0000618</td>
<td>0.000061</td>
</tr>
<tr>
<td>60 sec</td>
<td></td>
<td>0.0002658</td>
<td>0.000091</td>
<td>0.0000795</td>
<td>0.0000486</td>
<td>0.000034</td>
<td>0.000031</td>
</tr>
</tbody>
</table>
Figure 3.8 Normalized CDF Plot of Divergence Estimate Error for HZA

Figure 3.9 Normalized CDF Plot of Divergence Estimate Error for MLA
3.4 Monitor Threshold

From the continuity allocation specified in chapter 2, we have $P(FA) = 10^{-7}/15$ sec for all SVs. Given a maximum of 10 critical satellites (SVs), as prescribed by the LAAS MASPS, we then have $P(FA) = 10^{-8}/15$ sec/SV. It was shown above that the test statistic is dominated by nominal ionospheric divergence, which is essentially constant over 15 sec intervals, so all 2 Hz tests in any 15 sec interval are mutually dependent. Therefore the required $P(FA) = 10^{-8}$/test. The associated gaussian fault-free detection multiplier for this probability is $k_{ffd} = 5.73$. The resulting monitor threshold in equation (3-4) is $T_c = k_{ffd} \times \sigma_d = 5.73 \times 0.004 \text{ m/s} = 0.0229 \text{ m/s}$.

In summary the CCD monitor developed in this chapter is simple and easy to implement as it uses two first order filters in series with a recommended time constant of 25 seconds with a simple detection function. For simplicity both filter time constants are equal. It is shown from data analysis that the contribution due to filtered code noise is a order less than the contribution due to ionosphere to fault free standard deviation of divergence rate. Ionospheric divergence analysis is presented using LTP data and WAAS ‘supertruth’ data, and the results are very close. In the next chapter we introduce the third phase of this research work describing the integrity analysis that uses the results from analysis on aircraft smoothing filters and CCD monitor design.
CHAPTER 4
MONITOR INTEGRITY ANALYSIS

The CCD monitor developed in the previous chapter is validated by integrity analysis. In literature we find the concept of Maximum Allowable Error in Range (MERR) used in integrity monitors for LAAS ranging sources. In [Zau02] we see that the probability of miss detection does not necessarily bound the probability of misleading information for error levels less than MERR. [Rif06] presents a time varying MERR formulation that addresses certain limitations associated with earlier static methods. But the analysis takes into account the time to alert as it strongly impacts the MERR magnitude for signal deformation fault modes. In this chapter we present a new direct approach for computing the overall probability of Loss of Integrity (LOI). The analysis eliminates the need for a time to alert requirement for CCD monitors and simplifies the analysis of transient errors from satellite-based faults.

4.1 Integrity Analysis Fundamentals

Following the integrity definitions in the LGF specification and their interpretations in reference [Cas06], the probability of a Loss of Integrity (LOI) due to a space segment failure event on satellite \( k \) is defined for Category I as,

\[
P(\text{LOI} \mid \text{fault}_k) \equiv P\left( \left| e_p(\tau) \right| > VPL(\tau) \forall \tau \in [t, t + TTA] \right. \\
\left. \cap \left| q_k(\tau) \right| < T_{ccd} \forall \tau \in [-\infty, t + TTA - \tau_g]\right) \mid \text{fault}_k
\]

where, \( TTA \) is the specified Time-To-Alert (3 sec) and \( \tau_g \) is the delay between the arrival time of the GPS signal at the LGF and the time at which last bit of integrity data based on
that signal is broadcast to users by the LGF. For the ground system, $\tau_g$ will not exceed 1.5 sec. Therefore $TTA - \tau_g = 1.5$ sec.

For Category I, only vertical errors are considered because the impact of exceeding the protection level is more severe for the vertical case than for the lateral. This is true because $VPL$ is nearly always greater than $LPL$ while the alert limit is much more stringent in the vertical ($VAL = 10$ m) than the lateral ($LAL = 40$ m). In contrast, for DCPS only horizontal errors are of interest. For clarity, this development will detail the Category I case. The development for the DCPS case is essentially the same. The results for DCPS will be given at the end of the Category I analysis.

Strictly, a vertical error $e_v$ can lead to $LOI$ only if it exceeds both $VPL_{H0}$ and the ephemeris error bound $VPL_E$. In this analysis, for simplicity and conservatism, we assume that exceeding $VPL_{H0}$ alone is a sufficient condition to lead to possible $LOI$.

To meet the Category I integrity risk allocation for the CCD monitor ($P_a$), it must be shown that

$$P(LOI \mid fault_k) < P_a \text{ for } t \in [\text{any 150 sec interval}], \quad (4-2)$$

where, $P_a = 10^{-4}$. In the integrity analysis that follows, the most threatening time interval is covered by assuming the theoretical worst-case satellite geometry. In addition, all values of $t$ surrounding the onset of a CCD failure event are considered in the analysis. Therefore the 150 sec interval requirement is implicitly satisfied.

Assuming statistical independence of random noise (i.e., non-failure) contribution to the position error and test statistic
\[ P(\text{LOI} \mid \text{fault}_k) = P(\{ e_k(\tau) \mid VPL(\tau) \forall \tau \in [t, t + TTA] \mid \text{fault}_k \} ) \cdot \]
\[ P(\{ q_k(\tau) \mid T_{\text{mon}} \forall \tau \in [\infty, t + TTA - g] \mid \text{fault}_k \} ) \]

Define \( \tau_e \) to be any individual time in the interval \([t, t + TTA]\) and \( \tau_q \) to be any individual time in the interval \([t, t + TTA - g]\). Then it is true that,

\[ P(\text{LOI} \mid \text{fault}_k) \leq P(\{ e_k(\tau_e) \mid VPL(\tau_e) \mid \text{fault}_k \} ) P(\{ q_k(\tau_q) \mid T_{\text{mon}} \mid \text{fault}_k \} ) \quad (4-4) \]

Because \( \tau_e \) and \( \tau_q \) can be chosen arbitrarily within their respective intervals, it is permissible to choose each time to separately minimize each of the two probabilities on the right-hand-side of (4-4). One straightforward choice is to select:

\[ \begin{align*}
\tau_e &= t \\
\tau_q &= t + TTA - g
\end{align*} \]

So, that the monitor detection probability (second term on the right-hand side of (4-4) is evaluated \( TTA - g = 1.5 \) sec later than the position error bounding probability (first term on right hand side of (4-4)). The basis for this choice is that the monitor has 1.5 seconds of extra time (relative to the position error) for the test statistic to grow beyond the threshold.

Consider now the two terms on the right-hand side of inequality (4-4) separately. For brevity in notation in the following few steps, the time variables will be temporarily omitted. They will be included again in the final result.

Given a failure on satellite \( k \) with a resulting \( b_k \) not close to zero, the first term can be simplified as a one-sided probability as follows:
\[ P( | e_v | > VPL \mid fault_k ) \approx P( | S_{v,k} b_k | + \sum_{i=1}^{n} S_{v,i} v_i > k_{ffmd} \sigma_v ) \]
\[ = 1 - Q_{\sigma_v} \left\{ k_{ffmd} \sigma_v - | S_{v,k} b_k | \right\} \]
\[ = 1 - Q \left\{ k_{ffmd} - \frac{| S_{v,k} b_k |}{\sigma_v} \right\} \]
\[ = 1 - Q \left\{ k_{ffmd} - \frac{\sigma_{v,k}}{\sigma_v} | b_k | \right\}. \]

Figure 4.1 LGF Integrity Risk Given Fault on Ranging Source \( k \)

The worst-case probability occurs when \( \frac{\sigma_{v,k}}{\sigma_v} \to 1 \). Therefore, it is convenient to conservatively use the following satellite-geometry-free expression

\[ P( | e_v | > VPL \mid fault_k ) = 1 - Q \left\{ k_{ffmd} - \frac{\sigma_{v,k}}{\sigma_v} | b_k | \right\} = Q \left\{ \frac{\sigma_{v,k}}{\sigma_v} | b_k | - k_{ffmd} \right\} \] (4-6)

Using the same method, the second term on the right-hand side of equation (4-4) can be simplified as follows:

\[ P( | q_k | < k_{ffid} \sigma_d \mid fault_k ) \approx P( | r_k | + \eta_k < k_{ffid} \sigma_d \mid fault_k ) = Q \left\{ k_{ffid} - \frac{| r_k |}{\sigma_d} \right\} \] (4-7)

Substituting (4-5), (4-6), and (4-7) into equation (4-4) yields,

\[ P(LOI \mid fault_k) = Q \left\{ \frac{| b_k(t) |}{\sigma_k(t)} - k_{ffmd} \right\} \times Q \left\{ k_{ffid} - \frac{| r_k(t + \Delta t) |}{\sigma_d} \right\} \] (4-8)
where, $\Delta t \equiv TTA - \tau_g = 1.5$ sec.

In general, a space segment failure event on satellite $k$ will cause different transient responses in the differential position error $b_k$ and the monitor test statistic $r_k$. The LOI probability in equation (4-8) will be a function of both of these failure response functions, the failure magnitude, the elapsed time since failure onset, and the ground and airborne receiver tracking start times. For every type of satellite failure it is necessary to find the conditions that maximize the $LOI$ probability and to determine whether the result exceeds the integrity risk allocation $P_a$ for the failure mode.

4.2 Integrity Analysis - CCD Monitoring

4.2.1 Differential Ranging Error. Divergence-related differential-ranging error is due to two separate sources:

- *Differences in ground and air filter implementations* will cause differential ranging error in response to divergence.

- *Differences in start times of ground and air filters* will cause transient differential error even if the avionics implements the same filter as the LGF.

Given a divergence failure with CCD rate $d$ and time of onset $t = 0$, differential ranging error exists only when both filters are tracking ($t > t_{0a}$ and $t > t_{0g}$) and after onset of the failure ($t > 0$). Therefore the differential ranging error, $b_k$ in the general analysis in the preceding section, can be expressed as

$$b_k(t) = \frac{d}{d_{nom}} \left| e_g(t - \max[t_{0g}, 0]) - e_a(t - \max[t_{0a}, 0]) \right|. \quad (4-9)$$
Note that \( t_{0g} \) and \( t_{0a} \) can be negative, signifying possible filter start times prior to failure onset. The standard deviation of the ranging error used at the aircraft, \( \sigma_k \) in the preceding section, is

\[
\sigma_k(t) = \sqrt{\sigma_{pr_{-}gnd,nom}^2 + \sigma_{pr_{-}gnd}^2 + \sigma_{pr_{-}air,nom}^2 + \sigma_{div_{-}gnd,}^2 + \sigma_{div_{-}air}^2}. \tag{4-10}
\]

As \( d_{nom} = 0.018 \) m/s is prescribed by the LAAS MOPS and LGF Specification to be used as the standard deviation of nominal ionospheric divergence rate, then \( \sigma_{div_{-}gnd} = e_g(t - t_{0g}) \) and \( \sigma_{div_{-}air} = e_a(t - t_{0a}) \). Therefore, (4-10) can be rewritten

\[
\sigma_k(t) = \sqrt{\sigma_{pr_{-}gnd,nom}^2 + \sigma_{pr_{-}gnd,nom}^2 + e_g^2(t - t_{0g})^2 + e_a^2(t - t_{0a})^2}, \tag{4-11}
\]

and using (3-5) we then have

\[
\frac{b_k(t)}{\sigma_k(t)} = \frac{d}{\sigma_{nom}^2} \frac{\max\{t_{0g} - 0, 0\} - \max\{t_{0a} - 0, 0\}}{e_g^2(t - t_{0g})^2 + e_a^2(t - t_{0a})^2}. \tag{4-12}
\]

Equation (4-12) may be substituted into the right-hand side of equation (4-8) for the CCD case. Ideally, the LGF monitor performance should be independent of \( \sigma_{pr_{-}gnd,nom} \) and \( \sigma_{pr_{-}air,nom} \), which may vary over time and location. In the most conservative analysis, it could be assumed that \( \sigma_{pr_{-}gnd,nom} = \sigma_{pr_{-}air,nom} = 0. \) However, the LAAS MOPS ensures that \( \sigma_{pr_{-}air,nom} > 0.13 \) m. In this analysis, it is therefore assumed that \( \sigma_{pr_{-}gnd,nom} = 0 \) and \( \sigma_{pr_{-}air,nom} = 0.13 \) m.

The LGF filter to be implemented is a first order digital LTI filter with a 100 sec time constant (as defined in the LGF Specification). LGF does not broadcast corrections for satellites during the first 200 sec of filtering, so only \( t > t_{0g} + 200 \) sec is considered in this integrity analysis.
For the aircraft, it is assumed a first order digital LTV filter is implemented. This filter differs from the LGF filter only during the first 100 sec of operation, when the effective filter time constant increases uniformly in time up to the 100 sec limit. (The revised MOPS will also permit the use of an LTI implementation, but it is a more benign case with respect to CCD so it is not considered explicitly here). It is also assumed that the aircraft will use filtered measurements immediately (i.e., \( t > t_{0a} \)). The two digital filter responses to nominal divergence, \( e_g \) and \( e_a \), are plotted as functions of time in Figure 4.2.

![Figure 4.2 Ground (LTI) and Air (LTV) Filter Responses (Relative to Steady State) for Divergence Rate Input \( d_{nom} = 0.018 \text{ m/sec} \)](image)
4.3 CCD Monitor

As described in chapter 3, the LGF divergence monitor consists of two components: divergence rate estimator and a detection test. The input to the divergence rate estimator, $z$, is the raw code minus carrier measurement. The divergence rate estimator differentiates and filters the input $z$ to produce the estimator the output $d_2$.

Given a unit ramp divergence input, the noise-free time response of the estimator $d_2(t) = u_2(t)$ is shown in Figure 4.2. As the divergence estimator is a linear filter, the amplitude of the time response for other divergence inputs will scale linearly with $d$.

The test statistic in equation (4-8) is the estimator output for a failed satellite $k$:

$$ r_k (t + \Delta t) = d \cdot u_2 (t + \Delta t - \max [t_{0_{g}}, 0]) $$

Equation (4-13) is substituted into the right-hand-side of equation (4-8) together with $\sigma_d = 0.004 \text{ m/s}$ and $k_{f, d} = 5.73$. [Recall that the monitor threshold $T_{c, d}$ (0.0229 m/s) is equal to the product of $\sigma_d$ (0.004 m/s) and $k_{f, d}$ (5.73)].

![Figure 4.3 Divergence Rate Estimator Response to Unit Ramp Divergence Input](image)
4.3.1 Loss of Integrity Probability given CCD Fault (Category I). Combining equations (4-8), (4-12), and (4-13), we obtain

\[ P_{LOI|\text{fault}}(t, t_{0g}, t_{0a}, d) = P_{\text{err}}(t, t_{0g}, t_{0a}, d) P_{\text{mon}}(t, t_{0g}, d) \]  

(4-14)

where,

\[ P_{\text{err}}(t, t_{0g}, t_{0a}, d) = Q\left\{ \frac{d}{d_{\text{nom}}} \left[ e_{g}(t - \max[t_{0g}, 0]) - e_{a}(t - \max[t_{0a}, 0]) \right] \right\} \]

\[ \times \sqrt{\sigma_{p_{\text{gnd,nom}}}^2 + \sigma_{p_{\text{air,nom}}}^2 + e_{g}(t - t_{0g})^2 + e_{a}(t - t_{0a})^2} - k_{\text{ffind}} \]  

(4-15)

\[ P_{\text{mon}}(t, t_{0g}, d) = Q\left\{ k_{\text{fid}} - \frac{d \cdot u_{z}(t + \Delta t - \max[t_{0g}, 0])}{\sigma_{d}} \right\} \]  

(4-16)

The new time-explicit notation \( P_{LOI|\text{fault}}(t, t_{0g}, t_{0a}, d) \) is introduced in place of the original \( P(LOI | fault_{k}) \). Note also that the satellite index \( k \) is no longer needed because the failure has already been applied to the worst-case satellite.

To demonstrate that the monitor performance is sufficient, it is necessary to directly evaluate \( P_{LOI|\text{fault}}(t, t_{0g}, t_{0a}, d) \) using equations (4-14)-(4-16) and show that it never exceeds \( P_{a} \), as required by condition (4-2). Unfortunately the function \( P_{LOI|\text{fault}}(t, t_{0g}, t_{0a}, d) \) is difficult to visualize graphically because four independent variables are involved. As an example Figure 4.4 shows a three-dimensional color-coded contour plot to describe the function. The plot corresponds to a single value of \( t_{0g} (-500 \text{ sec}) \) and shows \( P_{LOI|\text{fault}} \) as a function of \( t, t_{0a}, \) and \( d \). The value of \( P_{LOI|\text{fault}} \) is indicated by color. For ease in visualization, in the ranges of \( t \) and \( t_{0a} \) are limited to those that lead to the highest values of \( P_{LOI|\text{fault}} \).
The value $t_{0g} = -500$ sec is chosen because the largest values of $P_{LOI|\text{fault}}$ occur when the ground filter starts well before the failure ($t = 0$). This true because the mitigating effect of $\sigma_{\text{div,gnd}}$ (i.e., $e_g(t - t_{0g})$ in equation (4-15)) is negligible when the ground filter starts much earlier than the failure onset. Furthermore the 200 sec waiting period after filter start (implemented at the LGF), during which a failure can exist without danger to the aircraft, has already expired for any value of $t_{0g}$ smaller than $-200$ sec. An interesting point to note from Figure(4.4) is that as the fault magnitude ($d$) increases $P(LOI|\text{fault}_k)$ is worst when $t_{0a}$ is small and time ($t$) approaches $t_{0a}$.

Figure 4.4 Three-Dimensional Contour Plots of $P_{LOI|\text{fault}}$: Category I Case
Given the worst-case early start time for the ground filter, it is also evident that the maximum values of $P_{LOR|\text{fault}}$ occur when $t$ and $t_{0a}$ are positive and small. This is true because $P_{\text{mon}}$ is highest when $t$ is near zero, while $P_{\text{err}}$ is largest when $t_{0a} > 0^*$.  

The divergence onset of magnitude $d$ is defined to occur at time $t = 0$. For the purpose of interpreting the integrity analysis results, we will refer to the first term on the right-hand side of equation (4-8) as $P_{\text{ev|fault}}(t, t_{0a}, t_{0g}, d)$ and the second term as $P_{\text{md|fault}}(t - \max [t_{0g}, 0], d)$.

![Figure 4.5 Illustration for $P_{(\text{ev|fault})}$ and $P_{\text{md|fault}}$](image)

* Note from equation (4-15) and Figure 4.2 that $P_{\text{err}} \to 0$ as $t_{0a} \to 0$. 
If the ground filter starts close in time to the aircraft filter, then the monitor response for the first 200 seconds is not utilized for computation of $P_{md|fault}$ because the LGF does not broadcast corrections during this time. Hence $P_{md|fault} = 0$ during the first 200 sec. Therefore, as illustrated in Figure 4.5 the worst-case $P(LOI|fault)$ occurs when the ground filter has started well before the aircraft filter: theoretically speaking, when $t_{0g} = -\infty$ and aircraft filter has just started.

![Figure 4.6](image)

Figure 4.6 Worst-case $P_{LOI|fault}$ for any value of $t$, $t_{0g}$, and $t_{0a}$ versus Magnitude of Divergence Rate Failure: Category I Case

Figure 4.6 shows the maximum value of $P_{LOI|fault}$ for any value of $t$, $t_{0g}$, and $t_{0a}$ as a function of $d$. The figure shows that the peak value of $P_{LOI|fault}$ occurs for $d$ between 1 and 2 m/s and that the peak probability is below the allocated integrity risk requirement $P_a = 10^{-4}$. 
Per approach averaging was not used to validate monitor performance in this analysis. It was shown that the monitor meets the allocated integrity risk requirements for any individual approach by directly evaluating performance under the worst possible conditions of satellite geometry, air and ground filter start times, and failure magnitude. However statistical data of nominal ionospheric divergence rates was used to set the monitor threshold. This data was collected on different days and at different locations to provide a diverse sample set.
CHAPTER 5

CONCLUSIONS

GPS though initially developed by the DoD, has over the years found widespread applications in the civil society, more so in the civil aviation sector. Its reliability and accuracy is greatly enhanced by FAA’s implementation of DGPS in its architecture for the Local Area Augmentation System (LAAS). LAAS has strict requirements regarding accuracy, integrity, continuity and availability. Each of these in their own aspects have become a part of GPS research over the years for system performance enhancements based on varying applications of GPS.

In this research we have addressed the issue of code carrier divergence fault as a result of failure occurring at the satellite and developed a monitor to detect abnormal divergences. In this work we have presented a monitoring and integrity risk analysis for code-carrier divergence that is currently not in place at the LAAS Ground Facility (LGF). We suggest a 1st order LTV smoothing filter as a good choice for implementation at the aircraft and focuses the analysis on this implementation, but as other higher filter implementations are also possible the analysis approach was designed to be easily extendable to different filter implementations at the aircraft.

The designed CCD monitor is to be implemented at the LAAS ground facility. This monitor uses two 1st order LTI filters in series for divergence rate estimation, which is followed by a simple detection test. We present nominal values for monitor filter time constants and compute the fault free test metric based on experimental data.

A new direct approach is presented to compute the probability of Loss of Integrity for a space segment failure. This new method is applied it to the CCD monitoring
problem. Results from integrity analysis validate the monitor performance with probability of loss of integrity always less than $10^{-4}$ for category I aircraft precision approach. Different aircraft and ground filter start times are explicitly accounted for and results show that LAAS integrity requirements are satisfied.
APPENDIX
This appendix contains intermediate data analysis results obtained using archived field test data.

– ASCII data from Multipath Limiting Antenna (MLA) and High Zenith Antenna (HZA): pseudorange, carrier phase, C/No, and elevation.

– Binary OEM 4 dual frequency data to remove ionospheric divergence (‘iono’).

MLA data measurements below 5-degree elevation were not used. The following plots are shown for the HZA and MLA in appendix A and B respectively.

- Divergence rate estimate vs. time
- Divergence estimate error vs. time constant
- Elevation vs time
- Signal to noise ratio (C/N₀) vs. time
- C/N₀ vs. Elevation
- Divergence estimate error vs. Elevation
- Divergence estimate error vs. C/N₀
APPENDIX A

HZA RESULTS
Figure A.1 Example HZA Divergence Estimate

Figure A.2 HZA Divergence Estimate Error vs Time Constant
Figure A.3 Example Elevation vs Time

Figure A.4 Example C/N₀ vs. Time
Figure A.5  C/N₀ vs. Elevation for 3 SVs: 13, 28, and 31

Figure A.6  Divergence Estimate (Absolute values) Vs. Elevation

Figure A.6  Divergence Estimate Error vs Elevation for 3 SVs: 13, 28, and 31
Figure A.7  Divergence Estimate Error vs C/N0 for 3 SVs: 13, 28, and 31
APPENDIX B

MLA RESULTS
Figure B.1  Example MLA Divergence Estimate

Figure B.2  MLA Divergence Estimate Error vs Time Constant
Figure B.3  Example C/No vs Time

Figure B.4  Example Elevation vs. Time
Figure B.5 C/N₀ vs. Elevation for 3 SVs: 13, 28, and 31

Figure B.6 Divergence Estimate Error vs. Elevation for 3 SVs: 13, 28, and 31
Figure B.7 Divergence Estimate Error vs. C/N₀ for 3 SVs: 13, 28, and 31
BIBLIOGRAPHY


[ICD01] “GNSS Based Precision Approach Local Area Augmentation System (LAAS) Signal-In-Space Interface Control Document (ICD)”, RTCA., DO-246B. November 28, 2001


[DID08] “Data Item Description: Algorithm Description Document,” Federal Aviation Administration, LGF-SA-008.


